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# Comparing multi-scale cracking mechanisms in man-made composites and natural materials

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Man-made composites, like reinforced and fiber-reinforced concrete elements, behave similarly to natural composite structures (e.g., brittle rocks lying on a ductile substratum). In both cases, the cracking phenomenon of brittle layers depends on the scale of observation and is ruled by the Golden Ratio. Thus, a unique size-effect relationship, herein called Golden Scaling Law (GSL), is introduced and used to predict the crack pattern of concrete and rock structures. In accordance with several experimental data, GSL permits to calculate the values of crack width and crack spacing when the geometry of crack patter is known at a lower scale. Moreover, GSL can be applied to large composite structures without knowing the material performances, but by testing prototypes of lower dimensions.



## 1. Introduction

Depending on the scale of the structures under consideration, many engineered and natural materials show different mechanical behavior. Thus, size effect theories, based on a multiscale approach, analyze the intrinsic (due to microstructural constraints, e.g., grain size) and extrinsic effects (caused by dimensional constraints), in order to improve the knowledge of structural behavior in the field of material science and applied mechanics [1]. Nevertheless, several nonlinear problems of structural mechanics cannot be solved by conventional approaches, because of the complexity and uncertainty on materials properties, especially in composites at different scales [2,3].

For instance, in some structures made with brittle materials (e.g., concrete) and ductile reinforcement (e.g., steel rebar and/or fibers) [4], it is of particular interest to investigate not only the size effect on strength, but also the cracking phenomenon at different scales [5]. Indeed, as crack spacing and crack width increase with the structural dimension [6], the durability of large structures and infrastructures can be severely undermined by a heavy crack pattern.

Other crack patterns, related to natural composite structures, are generally associated with the scale of observation. In Structural Geology, for instance, it has long been established that lithology and thickness of brittle rocks, lying on ductile substrates, influence the spacing between the fractures on the earth's crust [7]. Similarly to concrete structures, fracture spacing and width increase with layer thickness and their estimation is of great importance in petroleum and gas industry, as well as in hydrology and waste management [8].

Several models were developed in the last decades for analyzing the cracking phenomenon of brittle or quasi-brittle composites at different scales. Most of the studies were carried out to introduce theoretical and semi-empirical formulae for the evaluation of crack width and spacing in the case of concrete structure [9] and rock beds [10]. Although the experimental investigations reveal similar behavior in man-made and natural composites, a unified approach for analyzing the crack pattern cannot be found in the technical literature. The authors believe that the size-effect law presented in the following sections, and based on the Golden Ratio, $\phi = 1.61803\ldots$, addresses that research gap for the first time.

## 2. Modelling the crack pattern of composites due to bond-slip

Regardless of the scale of observation, the cracking

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phenomenon of reinforced concrete structures and rock materials can be modeled in the same manner, i.e., by means of nonlinear fracture mechanics applied to a brittle material lying on a ductile substratum [10,11]. Cracks are due to the strain incompatibility of these adjacent layers, which in turn depends on the different strength and deformability of the materials. The free-body diagram of the composite structure, subjected to tensile load $N$, is illustrated in Fig. 1. The tensile load elongates elongates both the layers, but cracks occur only in the brittle material.

To model this phenomenon, a series of equilibrium and compatibility equations have to be written in accordance with the free-body diagram shown in Fig. 1. In a generic cross-section of the composite, stresses in the brittle ($\sigma_b$) and ductile material ($\sigma_d$) yield the following equilibrium equation:

$$N = \sigma_b A_b + \sigma_d A_d \tag{1}$$

where, $A_b$ = cross-sectional area of the brittle layer; $A_d$ = cross-sectional area of the ductile layer.

Bond stresses $\tau$ at the interface of the layers are in equilibrium with the state of stress acting in the ductile substratum:

$$\frac{\partial \sigma_d}{\partial z} = \frac{\tau}{A_d} l \tag{2}$$

where, $z$ = horizontal coordinate; and $dz \cdot l$ = interface area between the two materials.

The values of slip $s$ on the interface must be compatible with the strains of both the ductile and brittle materials, as stated by following equation:

$$\frac{\partial s}{\partial z} = \varepsilon_d - \varepsilon_b \tag{3}$$

where $\varepsilon_d$ and $\varepsilon_b$ = strain in the ductile and brittle layer, respectively.

The magnitude of the bond stress depends on the values of slip, in accordance with the so-called bond-slip model ($\tau$-$s$). Such model and the boundary conditions (generally defined by the cracks of the brittle layer) need to be introduced to solve the system of Eqs. (1)–(3), under the hypothesis of linear elastic behavior of the two materials.

In the case of reinforced concrete structures, with and without fibers (i.e., a hybrid reinforcement), these equations constitute the “tension stiffening” problem, whose solution provides the geometry of crack pattern [11]. For instance, the numerical procedure proposed by Fantilli et al. [5] for the solution of Eqs. (1)–(3) gives crack width, crack depth and crack spacing in reinforced concrete beams and ties.

The above geometrical properties of the cracking phenomenon depend on the scale of observation, similarly to the tensile strength of brittle and quasi-brittle materials [12]. In other words, the following size effect law, with the form of a power law [13], can be introduced:

$$\frac{cs}{cs_0} = \frac{w}{w_0} = \left(\frac{D}{D_0}\right)^{\alpha} \tag{4}$$

where $D_0$ = reference dimension; $D$ = dimension at a generic scale; $cs_0$, $cs$ = crack spacing at reference dimensions and at generic scale, respectively; $w_0$, $w$ = crack width at reference dimensions and at generic scale, respectively; and $\alpha$ = exponent that has to be defined by fitting experimental and/or numerical data.

According to the results of the numerical procedure introduced by Fantilli and Chiaia [5], when the geometrical dimensions of the reinforced concrete members are doubled (i.e., $D = 2\,D_0$) the distances between the cracks, and the maximum crack width as well, increase of a factor $\phi = 1.61803\ldots$

In mathematics, $\phi$ is known as the Golden Ratio, i.e. the “divine proportion”. A first definition of the Golden Ratio was proposed by Euclid (300 b C.). In physics, this number, intimately interconnected with the Fibonacci sequence (1, 2, 3, 5, 8, 13…), controls growth in several natural patterns. In fact, the limit of the ratios of two successive terms of the series tends to $\phi$ [14].

As the Golden Ratio also recurs in the size effect law of crack pattern when $D = 2\,D_0$, the exponent $\alpha$ in Eq. (4) can be computed as follows:

$$\alpha = \frac{\log 2}{\log \phi} \cong 0.7 \tag{5}$$

Hence, a new size effect model, called Golden Scaling Law (GSL) and described by Eq. (4) – with $\alpha = 0.7$, is herein introduced for the analysis of crack patterns in solid materials.

## 3. Comparison with experimental data

The capability of GSL to predict crack spacing and crack width in man-made composites (i.e., reinforced concrete and fiber reinforced concrete), and in natural structures (i.e., superimposed layers of rocks), can be proven by considering the results of some experimental campaigns performed at different scales.

### 3.1. Traditional reinforced concrete and fiber-reinforced concrete ties and beams

Structural ties, made with a steel rebar surrounded by concrete

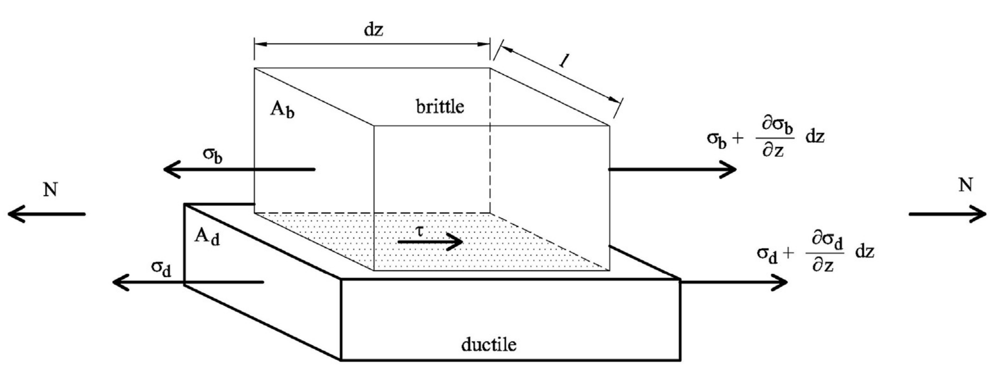


**Fig. 1.** Free-body diagram of a natural or man-made composite subjected to uniaxial tension.

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