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First versus second gradient energies for planar sheets with two families of inextensible fibres: Investigation on deformation boundary layers, discontinuities and geometrical instabilities

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A B S T R A C T

We present a homogenized model for the analysis of a 2D continuum with two straight families of inextensible fibres embedded in it. The kinematics of the continuum is analyzed and, motivated by phenomenological observations, it is assumed that the strain energy depends on the shear deformation of the fibres and on their bending curvature. It is shown that in order to account for the latter deformation it is necessary to introduce second gradient strains. The problem is formulated as a nonlinear constrained minimization, after introducing a suitable discretization of the domain. Some deformation processes are simulated using different constitutive hypotheses, comparing the predictions obtained assuming the presence of only first gradient or second gradient deformations, or a combination of both. It is found that the first gradient model leads to the presence of discontinuities in the rotation of the fibres, while the second gradient model regularizes these discontinuities by means of boundary layers. In particular in some deformation processes an instability of geometrical nature is observed when the second gradient model is used, that can be suppressed by the first gradient contribution.

1. Introduction

The characterization of the mechanical behaviour of woven fabrics is of paramount importance in the design of mechanical elements and of the material itself, in order to take advantage of all the peculiar properties of the composite. With respect to traditional composites, woven fabrics present dimensional stability, balanced properties in the reinforcement plane, high impact resistance, good drapability, so that they are suitable to be formed as doubly curved components. In this work we refer to 2D fabrics, obtained interlocking two families of fibres. This is the most common type of fabric used in the engineering practice, and owes its mechanical properties to the interaction between the fibres. While extension in the fibre direction is almost negligible, due to the large elastic modulus of the fibres (often carbon, glass or other high performance materials are used), shear deformation, i.e. the ability of the fibres to undergo relative rotation, can usually be large [1]. The latter mode of deformation is mostly responsible of the easiness of the fabric to modify its shape and to adapt to the final form required in the design. Friction and slippage of fibres is also a major issue in plane fabrics, when the deformation becomes large. In this context, some significant studies are presented in Refs. [2,3].

Disregarding the latter effects, that introduce energy dissipation, a fabric characterized by shear deformation alone, under the action of external loads, would develop sharp variations of the rotation along lines, giving rise to kinks in the deformation pattern. Indeed this feature is exploited in simplified models for interpreting the results of experimental tests [4–6]. A careful examination of the deformation pattern, as done in Ref. [7] reveals, however, that such sharp kinks do not occur, since the fibres bear some finite flexural rigidity, so that bending is observed.

Apparently, shear and bending are the main deformation modes of woven fabrics, at least for plane deformation and for moderately large displacements. The overall response of the fabric depends on

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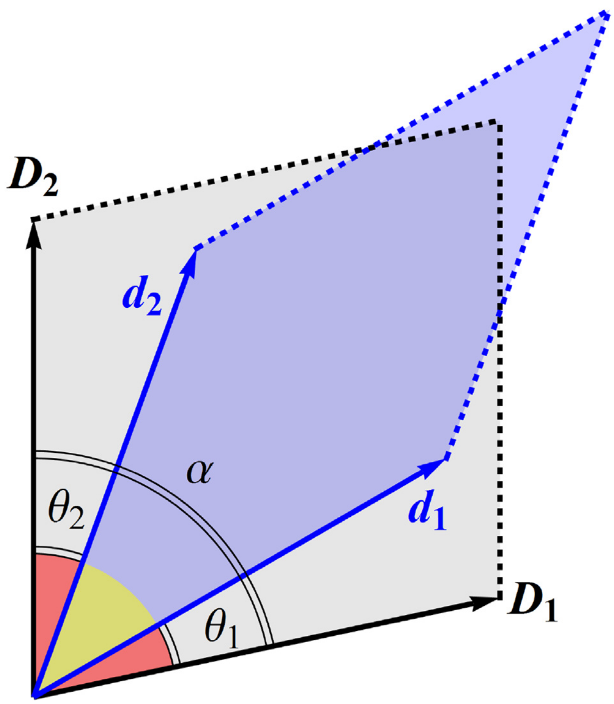

Fig. 1. Definitions of the rotation angles.

a competition between these two modes. If the shear stiffness prevails, sharp transition zones appear in the element, and eventually the deformation locks. On the contrary, if the bending stiffness prevails, a smoother deformation is obtained, but the element, or part of it, can present floppy modes.

In the work we analyze the relative influence of shear and bending stiffness on the plane deformation of a bidirectional tissue, disregarding all other mechanisms of deformation. Particularly, in addition to the dissipation mechanisms already mentioned, we disregard the out of plane displacement due to wrinkling, that occurs when some part of the element undergoes compression stresses. This aspect will be the object of future investigations, where the methodologies proposed among others in Refs. [8–10] will be employed.

The source of instability in the fabric is an important issue, and its origin needs to be investigated. In models proposed in the literature for the onset of instability the fabric is considered as a thin plate, and classical von Karman's theory is applied [11,12]. However, those studies do not consider the presence of direction of quasi inextensibility, nor the in plane bending mechanism of deformation. In the paper it will be analyzed the effect of the two deformation mechanisms (shear and bending) on the occurrence of in plane unstable deformation modes.

Testing methods for the mechanical characterization of fabric concentrate on the measure of the shear energy associated to the relative rotation between the fibres. Specific test have been designed to this end, like the direct shear test, the Picture Frame test, and especially the Bias Extension Test that is largely used in the engineering practice, see Refs. [13–20]. In Refs. [6,21] the latter test, and some modified versions of it, were examined, and it was shown that, in general, both shear and bending deformations influence the response of the specimen. According to the relative stiffness of the two modes of deformation, different response were obtained. However, the results in Ref. [6] were found on the basis of simplified kinematic assumptions, that, as will be shown in the present work, only hold when the deformation tends to a limit value. In general, a non uniform deformation state arises in the specimen. In this paper a more accurate analysis of plane deformation states of rectangular samples will be carried out.

The material will be considered as an homogenized continuum, at each point of which there are two inextensible directions, coinciding with the fibre directions. Two families of straight parallel fibres will be considered, according to the most common types of woven fabrics.

The material will be modeled as a second gradient continuum, identifying the first gradient deformation with the shear strain (extensional strains, if needed, could be added), while the bending deformation of the fibres will be identified with the second gradient deformation. In this way a reduced couple stress model is obtained. Analogous approaches can be founded in Refs. [4,7,22]. In this context the symmetry analysis presented in Refs. [23,24] can be very useful for characterizing the anisotropic properties of the model. Several homogenization approaches for macro model of woven fabrics can be found in Refs. [25–29] or in Refs. [30,31]. Alternatively micro-models as those proposed [32–34] can be used for the identification of the mechanical parameters.

Summarizing aim of the work is to investigate the relative influence of the shear and bending deformation on the response of

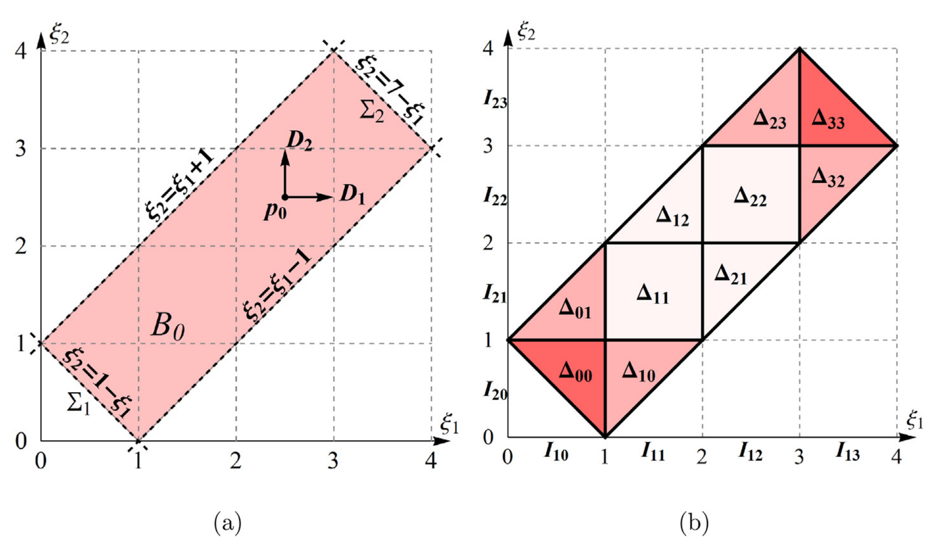

Fig. 2. Reference configuration: (a) Initial configuration and referential inextensible directors, (b) Partitioning and labeling of subregions.