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Free vibration of refined higher-order shear deformation composite laminated beams with general boundary conditions

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In this paper, a unified formulation which is based on a general refined shear deformation beam theory is presented to conduct free vibration analysis of composite laminated beams subjected to general boundary conditions. In the refined theory model, the displacement fields are chosen by including the high-order variation of transverse shear strain through the thickness of the beam and meeting the stress-free boundary conditions on both the top and bottom surfaces. With considering the material couplings and the Poisson's effect, the governing equations and appropriate boundary conditions are derived from the Hamilton's principle. Exact solutions are obtained by employing the method of reverberation ray matrix (MRRM). In order to implement general boundary conditions, the artificial spring boundary technique is introduced in the MRRM to make it suitable for different boundary cases. The present solutions are compared with those available in the literature to confirm their validity. A systematic parameter study for composite beams with various boundary conditions, fiber orientations, lamina numbers and orthotropic ratios is also performed. New results for free vibration involving composite laminated beams with various boundary constraints are also presented for the first time and they may be served as benchmark for researchers in this field.

1. Introduction

Due to the attractive properties in strength, stiffness and lightness, composite laminated structures are widely used in many engineering applications such as aircraft structures, space vehicles, turbo-machines, sports and other industrial applications as the fundamental structural elements. In these applications, one or more beam components are contained in almost every engineering structure. Besides, many structures can be modeled at a preliminary level as beams. It is known that the composite laminated beams are often applied in complex environments and subjected to various dynamic loads, the structures may fail and collapse because of material fatigue resulting from violent vibrations. Therefore, it's of increasing importance to have a thorough understanding of the structural vibrations and reduce them through proper design to improve the performance of composite laminated beams and take full advantage of them.

To obtain reliable and accurate results of the vibration characteristics for composite laminated beams, various beam theories have been proposed in the past few years. In the early stage, the classical beam theory (CBT) referred to as Euler-Bernoulli beam theory has been employed to predict the vibration behavior of slender laminated beams [1]. For moderately thick beams, the CBT underestimates deflection and overestimates natural frequency because it ignores the transverse shear deformation effect [2,3]. As a remedial measure, the CBT has been modified by taking the transverse shear deformation into consideration in beam bending, which results in the so-called first-order shear deformation beam theory (FBT). Since the FBT violates the stress-free boundary conditions on the top and bottom surfaces, a shear correction factor is needed to weaken the discrepancy between the assumed constant stress state and the actual one [2–5]. As has already been demonstrated, the shear correction factor has a noticeable influence on the accuracy of the FBT solutions. To eliminate the deficiency of the CBT and FBT, the higher order shear deformation beam theory (HBT) without the use of the shear correction factor has been developed, and it includes the higher order variation of axial displacement or both axial and transverse displacements

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along the thickness of the beam. Regarding the higher-order shear deformation theory, a number of models with different shear strain shape functions have been proposed. A brief assessment of several higher-order shear deformation theories can be found in the work of Wu and Chen [6]. Reddy [7] developed a third-order shear deformation theory accounting for parabolic distribution of the transverse shear strains through the thickness, and the exact closed-form solutions of symmetric cross-ply laminated plate were obtained by using the Navier approach. Touratier [8] presented a higher-order shear deformation plate theory for the bending, free vibration and buckling analyses of simply-supported composite plates and shallow shells, where the distribution of transverse shear was represented by a certain sinusoidal function. Soldatos [9] proposed a general two-dimensional theory for the transverse shear deformable plate, which accounted for an unlimited number of choices of through-thickness displacement distributions. Karama et al. [10] presented a multi-layer laminated composite structure model by using the exponential function to predict the mechanical behavior of composite structures. In addition, several refined higher-order shear deformation theories have been proposed by various author [11–17], where the so-called stretching effect is taken into consideration. Generally, these higher-order shear deformation theories involve higher-order stress resultants that are difficult to interpret physically and require considerably more computational effort. Therefore, such theories should be used only when necessary and there remains scope to develop an accurate theory which is simple to use. Recently, another refined higher-order shear deformation theories involving only four unknowns has been presented by Tounsi and his co-authors [18–24]. This theory model is based on the assumption that the in-plane and transverse displacements consist of bending and shear components in which the bending components have no contribution to shear forces and the shear components do not contribute toward bending moments. Meanwhile, the bending component of in-plane displacement is similar to that given by the classical plate theory (CPT), and the shear component gives rise to the higher-order variation of shear strains across the thickness. In such a way, shear stresses can be vanished on the top and bottom surfaces. Although the high-order shear deformation theories were initially developed for plates or shells, application of these theories to composite laminated beam is immediate. For example, Vo and Thai [25–31] presented various refined higher-order shear deformation beam theories and these beam models were found to be appropriate and efficient in investigating the static and dynamic properties of composite laminated and functionally graded beams. However, the establishments of unified and exact solutions of refined higher-order shear deformation beam theories for predicting the vibration behavior of composite laminated beams with general boundary conditions remains a challenging task and is the focus of the present study. The Carrera Unified Formulation (CUF) permits one to develop a large number of beam theories with a variable number of displacement unknowns by means of a concise notation and by referring to a few fundamental nuclei [32–36]. The refined higher-order beam theories can be easily implemented on the basis of the CUF, and the accuracy of a large variety of beam theories can be established in a hierarchical and/or axiomatic vs. asymptotic sense.

In addition to the aforementioned beam theories, a great quantity of analytical and computational methods have also been developed to conduct the free vibration analysis of laminated beams, such as the meshless method [37–39], the closed-form solution [40–43], transfer matrix method [44,45], dynamic stiffness method [46–48], differential quadrature method [49–52], and finite element method [53–57]. From the review, it can be clear revealed that the available study for free vibration behavior of composite laminated beams is far from complete. It appears that most of the existing solution methods are often only customized for a specific set of classical boundary conditions and this typically requires constant modifications of the solution procedures to be applicable to different boundary cases, which will result in very tedious calculations. On the topic of general boundary conditions, only the variational method [58–61] has been provided. Recently, a frequency-domain analysis technique named the method of reverberation ray matrix (MRRM) was presented by Pao [62] and Howard [63]. Due to its favorable adaptability, the MRRM has been applied in many composite structures for handling the dynamic problem [64–72]. However, as far as the authors know, there is no study available on vibration analysis of composite laminated beams with general boundary conditions by using the MRRM.

Motivated by the limitations in the research background, this paper aims to present a unified formulation to conduct the free vibration analysis of general refined higher-order shear deformation composite laminated beams with arbitrary lamination schemes and general boundary conditions. In the refined theory model, the axial and transverse displacements are composed of bending and shear components in which the bending components have no contribution to shear forces and the shear components do not contribute toward bending moments. Meanwhile, the shear component of axial displacement generates the higher-order variation of shear strain and hence shear stress through the depth of the beam in such a way that the shear stress vanishes on the top and bottom surfaces. With considering the elastic couplings coming from the material anisotropy and the Poisson's effect, the three governing equations and appropriate boundary conditions are deduced by applying the Hamilton's principle. The general boundary condition of the end of the beam is implemented by introducing three groups of linear springs and two groups of rotational ones. Accordingly, the MRRM is redefined to obtain exact solutions for different boundary cases. The accuracy of a variety of refined higher-order shear deformation beam theories to predict vibration behaviors of laminated beams is investigated. The effects of the elastic restraint parameters, layout schemes, lamina number and material anisotropy are also studied and reported.

2. Theoretical formulations

2.1. Description of the model

Consider a composite laminated beam of length L_x, width L_y and thickness h, as illustrated in Fig. 1. A Cartesian coordinate system (*o-xyz*) with the coordinates x along the central axis, y along the width direction and z along the thickness direction is used to describe the wave propagation in the beam. Thus, the deformations of the laminated beam are characterized by its center axis and take place in the *x-z* plane. At the two ends $x = 0$ and $x = L_x$, the general restraint condition is achieved by setting up three groups of translational springs ($k_{0,L_x}^{u_0}$, $k_{0,L_x}^{w_b}$ and $k_{0,L_x}^{w_s}$) and two groups of rotational springs ($K_{0,L_x}^{\phi_b}$ and $K_{0,L_x}^{\phi_s}$). These springs are continuously distributed along the width. By assigning the boundary springs with various stiffness values, different boundary forces can be simulated to be imposed on the ends of the beam [58–61,71,72]. In general, the beam is made up of some or many laminate layers; each consists of unidirectional orthotropic materials in arbitrary orientations with