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Effective shear modulus of solids reinforced by randomly oriented-/aligned-elliptic nanofibers in couple stress elasticity

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ABSTRACT

Nowadays, by adding a small amount (about 0.5–5% by weight) of a desired nanomaterial to a matrix having certain properties one may design a multifunctional nanocomposites with a remarkably improved macroscopic properties of interest. The capability of conventional continuum theories in treating the problems of embedded ultra-small inhomogeneity with any of its dimensions comparable to the characteristic lengths of the involved constituent phases is questioned, mainly, on the grounds of the accuracy and the size effect. The micromechanical framework based on the Eshelby's ellipsoidal inclusion theory [1] which has been widely used to estimate the overall behavior of composites falls under the same category, as is size insensitive. In this work, effort is directed at the prediction of the macroscopic shear modulus of composites consisting of nano-/micro-size fibers of elliptic cross-sections via couple stress theory, a physically realistic theory that encompasses the size effect. To this end, the fundamental equations of couple stress elasticity in elliptic coordinates are derived and several fundamental elliptic inhomogeneity problems in plane couple stress elasticity are solved analytically. For the purpose of the application of these results to the study of the effective properties of the composites of interest, Mori and Tanaka theory [2] is first reformulated in the mathematical framework of couple stress theory. Subsequently, the overall shear modulus of solids reinforced by aligned as well as randomly oriented elliptic nanofibers will be predicted. The influences of the size, shape, orientation, rigidity, and intrinsic length of the reinforcing nanofibers as well as the effects of the characteristic length of the matrix on the effective shear modulus of the composite are addressed.

1. Introduction

Nanomaterials are referred to materials that have at least one external dimension at the nanoscale, i.e. in the order of approximately 1*nm* to 100*nm*. Generally, thin films or surface coatings as well as two-dimensional materials, for example graphene and stanene have one dimension in the nanoscale and their other two dimensions are extended beyond. Materials with two dimensions in the nanoscale are categorized as fibers, for example nanowires, carbon nanotubes, and electrospun fibers. Materials having all three external dimensions at the nanoscale are generally referred to as nanoparticles and include quantum dots, colloids, precipitates, and nanocrystalline materials. Nanomaterials, due to their large surface-to-volume ratio and quantum effects can significantly enhance or alter such properties as reactivity, strength, optical, magnetic, and electrical characteristics. Nowadays, multifunctional nanocomposites of desired mechanical, optical, electrical, and magnetic properties may be achieved through incorporation of nanosize additives of specific characteristics into a matrix of standard material. For instance, addition of carbon fibers and bundles of multi-walled carbon nanotubes to polymers results in composites which are used to control or enhance conductivity. Some other types of nanomaterials with applications in nanocomposite technology are minerals, exfoliated clay stacks, and electrospun fibers. For example, plastics and nanosized flakes of clay are incorporated in fabricating car bumpers. For resistance to wear and damage as well as heat resistance purposes as in engines, appropriate nanocomposites are designed. It is noteworthy to point out that normally addition of only 0.5 to 5% by weight of nanomaterials is quite

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effective. Properties of the matrix material in the neighborhood of the reinforcements are significantly altered; properties vary continuously from that of the reinforcement at the matrix-reinforcement interface into the bulk of the matrix. Such reinforcements, due to their large surface-to-volume ratio bond to the surrounding matrix differently than the traditional additives. A principle outcome of the large amount of reinforcement-matrix interface area, which is typically an order of magnitude larger than that of ordinary composites, is that a rather small amount of nanosized reinforcement can lead to a remarkable effect on the macroscopic properties of the composite. Another feature of interest is that due to their extreme smallness, they are basically free of any defects and, moreover, can significantly strengthen materials by preventing free movements of matrix dislocations. It has been observed that some nanocomposites are 1000 times tougher than the bulk matrix materials. In the literature, carbon nanotube and graphene platelet have been used extensively as enrichment to obtain different types of composites with enhanced mechanical properties; see, for example, [3–6]. Lau et al. [7] have given a critical review on the interfacial bonding strength between nanotube enrichment and polymer matrix.

An immediate concern is to develop a method that is capable of estimating certain desired macroscopic mechanical properties of nanocomposites based on the relevant properties of their corresponding constituents with the dimensional spectrum ranging from a few nanometers to several millimeters. In the literature, the concept of size effect has been mainly associated to the effect of the change in sample size rather than the change in dimensions of its constituents. For instance, Lam et al. [8] have carried out some experimentations on the bending of micro-beams with thicknesses in the range of $20-115\mu m$, and emphasized that even for fixed length to thickness ratios, the deflections of the beams are different. While classical theory yields the same deflection as long as the ratio is fixed. They showed that, regardless of this ratio, nonclassical strain gradient theory leads to results that, in contrast to classical theory, are influenced by the size change and correspond to the experimental observations. Recently, the behavior of nanobeams has been examined using different augmented continuum theories. For example, Shodja et al. [9] obtained the exact closed-form solutions of the nanosized Bernoulli-Euler beam in the mathematical frameworks of first and second strain gradient theories. Barretta et al. [10] provided an Eringen-like model for Timoshenko nanobeams. Barretta et al. [11] introduced the first gradients of the axial and shear strain into the nonlocal theory of Eringen and Edelen [12], and subsequently treated functionally graded Timoshenko nanobeams. Torsional analysis of composite nanobeams, due to their important technological applications, has also increasingly received much attention in the past decade. For example, Pahlevani and Shodja [13] studied the surface and interface effects on torsion of eccentrically two-phase fcc circular nanorods by employing surface elasticity theory. Apuzzo et al. [14] proposed an enhanced nonlocal formulation for the torsional analysis of nanobeams. Zhu et al. [15] experimentation on fivefold twinned *Ag* nanowires with diameters ranging from 34 to 130*nm* reveals that Young's modulus, yield strength, and ultimate tensile strength of the nanowire increases with decreasing of the diameter. From a different angle, the phenomenon of the size effect can be associated with the microstructure/nanostructure of the material. In fact, the influence of the microstructure/nanostructure of different materials such as nanocrystalline materials and particulate-reinforced metal matrix composites has been confirmed experimentally by Kamat et al. [16], Hahn and Padmanabhan [17], Zhu et al. [18], and Zhong [19]. A further review of the experimental studies about the role of the size effect on the macroscopic mechanical properties of solids are given by Hemker and Sharpe Jr [20] and Greer and De Hosson [21]. In the realm of traditional continuum theories, capture of the size effect of the enrichments on the macroscopic behavior of solids is not possible. As far as this shortcoming is of concern, the micromechanical-based studies of the overall behavior of composites which utilize Eshelby's theory are no exceptions. In the pioneering work of Eshelby [1] the strain field, $\boldsymbol{\varepsilon}$ for the interior points of an ellipsoidal inclusion, Ω with a prescribed uniform eigenstrain field, $\boldsymbol{\varepsilon}^*$ is uniform and is given by $\boldsymbol{\varepsilon}=\mathbf{S}:\boldsymbol{\varepsilon}^*$ in which $\mathbf{S}$ is Eshelby's tensor. Owing to the fact that Eshelby's tensor, $\mathbf{S}=\mathbf{S}(\mathbf{C};\Omega)$ depends only on the shape of Ω and its elastic moduli tensor, $\mathbf{C}$, the induced strain and stress fields inside Ω are independent of the size of Ω . As it will be seen, this dilemma is circumvented in the context of the present work.

Previously, numerous investigators have utilized Eshelby's theory and provided commendable estimates of the effective elastic behavior of composites. For example, Nemat-Nasser and Taya [22], Nemat-Nasser et al. [23], and Iwakuma and Nemat-Nasser [24] employed Eshelby's theory to predict the effective moduli of solids containing nondilute distribution of noncoated ellipsoidal particles with the periodic microstructures. Furthermore, Shodja and Roumi [25,26] using Eshelby's equivalent inclusion method for multiinhomogeneities [27] presented a theory for the prediction of the overall properties of composites with the periodic distribution of multiphase reinforcements. A fairly thorough literature on this topic as well as other studies which are based on the size-independent Eshelby's inclusion problem is available in the review papers by Mura [28], Mura et al. [29], and Zhou et al. [30]. A critical shortcoming of such classical continuum approaches as Eshelby's, in addition to size independency, is in their inadequacy in providing a reasonably accurate solution in the vicinity of small defects and inhomogeneities. It is well-known that formulations within the mathematical framework of classical elasticity are bound to wave lengths and bodily dimensions large enough compared with the internal length-scales of the medium of interest. The early work of Cauchy [31] on the development of theory of elasticity suggested that stress at a given field point not only depends on the displacement of the point but also is influenced by the displacements of those in its neighborhood. Incorporation of this notion required resort to higher order continuum theories which can account for the discrete nature of materials. Based on this school of thought, strain gradient theories were developed and evolved in the literature. Consideration of the mathematical framework of a strain gradient theory, in general, leads to the introduction of one or more internal length scales which are inherently accountable for reflecting the discrete nature of the elastic bodies of interest. For example, for an isotropic elastic solid, formulation within first strain gradient theory as presented by Mindlin and Eshel [32], in addition to the two independent traditional Lame constants, involves two characteristic lengths, enabling the enhancement of the solution in the neighborhood of defects and inhomogeneities as well as accommodation for the size effect. Toupin and Gazis [33] in their study on surface effects and initial stress in continuum and lattice models of elastic crystals showed that the continuum description within first strain gradient theory is in agreement with the lattice model in which the nearest and next nearest interatomic interactions is accounted for.

The desire to increase the accuracy of solution in the vicinity of defects and inhomogeneities through accounting for the discrete nature and the structure of the atomic arrangements of matters, turned the attention of many prominent investigators towards the proposition of various higher order continuum theories, primarily, during the time period of about 1960 – 1975. For a fairly complete account of the historical development of the higher order continuum theories during the mentioned period, the readers may refer to the Introduction section in the recent work of Shodja et al. [34].