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Homogenisation of viscoelastic damping in unidirectional composites under longitudinal shear

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ABSTRACT

Explicit expressions for the complex longitudinal shear moduli of viscoelastic composites with arbitrary symmetric structure are obtained. Specific formulas of dissipative properties are obtained for tetragonal composites with elliptic and rectangular fibre cross-sections, as well as for hexagonal (not necessarily regular) composites with elliptic fibre cross-section. The correspondence principle and the duality rule are used. The results of calculations for particular cases agree well with the known published data.
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1. Introduction

One of the measures to reduce vibrations in structures is the use of the dissipative properties of structural materials [1,2]. Among the materials that can absorb vibration energy, polymeric materials and polymer-based composites are of interest. This capability is due to viscoelastic deformation, which, in a certain stress range, can be described by the linear theories [1,3–7].

Continuous models of micromechanics and homogenisation are able to give highly accurate results by solving boundary problems with numerical methods. E.g. in Refs. [8,9] the method of cells [10–12] is used and in Refs. [9,13] the finite element method (FEM) is applied. However, approximate mathematical models such as the rule of mixtures or more precise ones may be acceptable as well, their advantage is the simple formulas for the calculation of complex moduli of the.

The correspondence principle [14,15] is an effective tool for obtaining homogenisation formulas of complex moduli for viscoelastic composites. Such formulas are obtained from the formulas for the elastic properties after replacing the elastic constants by the relevant complex constants [16–19]. Thus, the quality of homogenisation of complex modules depends on the quality of correspondent elasticity models. The formulas for longitudinal shear moduli of orthotropic fibre composites with symmetrical structure [20] are of particular interest in this context.

$$\tilde{G}_{i3}^{(S)} = \int_0^1 \left[\int_0^1 \frac{d\chi_i}{G(\chi_1,\chi_2)} \right]^{-1} d\chi_{3-i} \tag{1}$$

(statically consistent or *S*-model) and

$$\tilde{G}_{i3}^{(K)} = \left[\int_0^1 \frac{d\chi_i}{\int_0^1 G(\chi_1,\chi_2) d\chi_{3-i}} \right]^{-1} \tag{2}$$

(kinematically consistent or *K*-model), where $G(\chi_1,\chi_2)$ – longitudinal shear modulus as a function of the non-dimensional coordinates on the unit square (3 denotes longitudinal direction). The advantages of these formulas are as follows.

A The universality. The composite structure, geometry of fibres and possible multicomponent nature are fully taken into

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account. Other known formulas either do not take into account geometry of a composite, or consider composites with fibres or particles of a simple shape.

B The formulas (1) and (2) give the lower and upper bounds of the shear moduli of orthotropic composites with a very narrow range of possible values [20,21]. The Voigt [22] and Reiss [23] bounds give too wide a range. Hashin-Shtrikman bounds [24] apply only to isotropic composites. Hill [25] and Hashin [26] estimates apply only to transversely isotropic composites. Besides, the above formulas do not take into account the composite structure, which is of importance not only for the elastic [27,28], but also for the dissipative properties [29].

C Good accuracy of averaging formulas (1) and (2) [20,21].

D These formulas can be simplified for most common cases of composite structure.

All this creates conditions for effective application of the correspondence principle to the formulas (1), (2). Such homogenisation of dissipative properties and comparison of its results with the known published data is the subject of this work.

The paper is organised in the following way. In Section 2, the problem of homogenisation for the complex shear moduli is formulated. In Section 3, the general homogenisation formulas and the duality rule are formulated. In Section 4, specific formulas of complex moduli for tetragonal composites with rectangular and elliptical cross-section fibres are derived. In Section 5, specific formulas of complex moduli for hexagonal composites with elliptical cross-section fibres are derived. Section 6 compares the results of numerical calculations by developed mathematical model with the results available in the literature.

2. Problem statement

Unidirectional continuously reinforced composite is subjected to a harmonic isothermal longitudinal shear. It is considered as a heterogeneous isotropic or transversely isotropic viscoelastic material. The heterogeneity is doubly periodic with periods $2a_1$ and $2a_2$ along the coordinates x_1 and x_2. Also, we assume that the heterogeneity has two systems of symmetry planes orthogonal to the axes x_1 and x_2.

The symmetry cell of unit thickness (along x_3), separated by two pairs of the nearest planes of symmetry, is the minimum representative cell (Figs. 1 and 2) whose integral stiffness properties correspond to macroproperties of the composite, i.e., to its effective characteristics.

According to the Neumann principle, this composite is orthotropic, and its main planes are parallel to the three symmetry planes of the structure. The composite is isothermally loaded with longitudinal shear in a monoharmonic way similar to (1):

$$\tilde{\tau}_{i3} = \tilde{\tau}^*_{i3}\exp(\boldsymbol{i}\omega t),\quad \tilde{\gamma}_{i3} = \tilde{\gamma}^*_{i3}\exp(\boldsymbol{i}\omega t). \tag{3}$$

Hereinafter, the macroscale values are marked by a tilde. Then the complex amplitudes of macrostresses and macrostrains can be expressed similar to (2):

$$\tilde{\tau}^*_{i3} = \tilde{G}^*_{i3}\tilde{\gamma}^*_{i3},\quad \tilde{\gamma}^*_{i3} = \tilde{Q}^*_{i3}\tilde{\tau}^*_{i3}. \tag{4}$$

Similar relations link complex microstresses and microstrains:

$$\tau^*_{i3} = G^*\gamma^*_{i3},\quad \gamma^*_{i3} = Q^*\tau^*_{i3}, \tag{5}$$

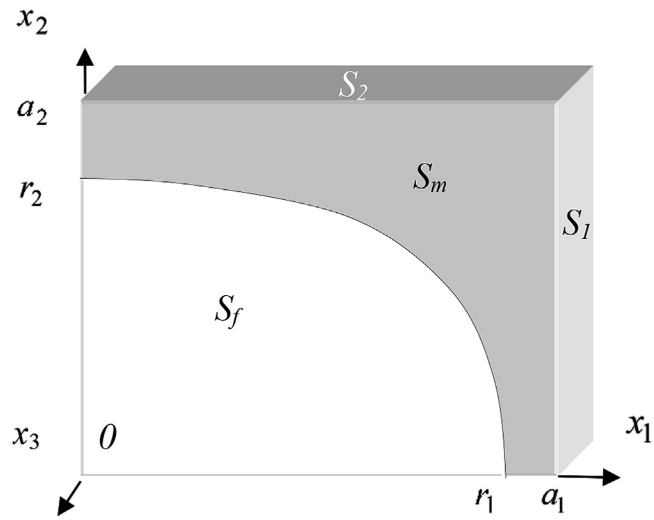

Fig. 1. Minimum representative cell of composite with tetragonal structure with fibre area S_f and matrix area S_m.

The complex shear stiffnesses G^*_{ij} and compliances Q^*_{ij} are of the form

$$G^*_{ij} = G'_{ij} + \boldsymbol{i}G''_{ij},\quad Q^*_{ij} = Q'_{ij} + \boldsymbol{i}Q''_{ij} \tag{6}$$

and are mutualy inverse:

$$G'_{ij} = Q'_{ij}\big/|Q_{ij}|^2,\ G''_{ij} = -Q''_{ij}\big/|Q_{ij}|^2,\quad Q'_{ij} = G'_{ij}\big/|G_{ij}|^2,\ Q''_{ij} = -G''_{ij}\big/|G_{ij}|^2. \tag{7}$$

The storage G'_{ij} and loss G''_{ij} moduli, as well as loss tangents

$$\eta_{ij} = G''_{ij}\big/G'_{ij} \tag{8}$$

are determined from experiments as functions of frequency and assumed to be known.

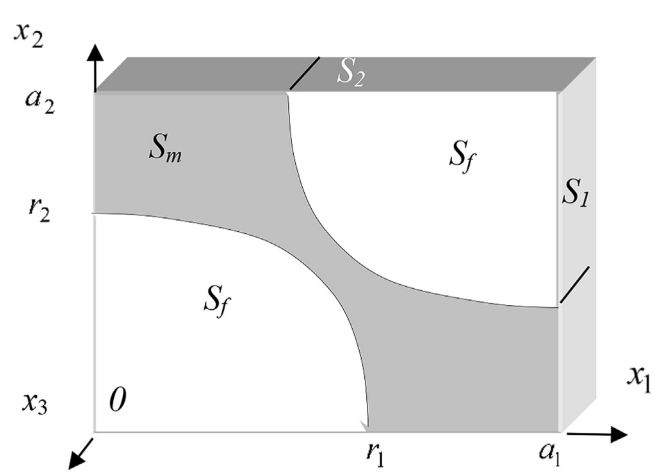

Fig. 2. Minimum representative cell of composite with hexagonal structure.

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