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Micromechanical modeling of thin composite and reinforced magnetoelectric plates – Effective electrical, magnetic, thermal and product properties

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ABSTRACT

Maxwell's equations and the equations of dynamic force and thermal balance are used to develop an asymptotic homogenization model for the analysis and design of magnetoelectric composite and reinforced plates. The developed model generates a set of unit cell problems the solution of which enables the determination of the effective coefficients of the homogenized structure. Of particular importance among the effective coefficients are the product properties which are manifested in the macroscopic structure but are not usually exhibited by the individual constituents. Examples of effective properties are the magnetoelectric, pyroelectric and pyromagnetic coefficients. The model is illustrated by means of several examples such as diagonally restrained magnetoelectric plates, plates reinforced with a triangular arrangement of ribs, wafer reinforced magnetoelectric plates and three-layered sandwich plates. From the nature of the structures considered, it is shown that it is more practical if a basic unit cell consisting of only a single arbitrarily oriented rib and a base plate is analyzed first. Then, the effective properties of magnetoelectric structures with more families of ribs can be obtained by superposition. It is shown in this paper that the effective coefficients as well as the field variables are functions of time and are not constants as predicted by other models. In fact, it is shown that other models are special cases of the model derived here when electrical conductivity is ignored and the aforementioned temporal functions are time-averaged by integrating them over the entire time spectrum. Thus, overall, this paper represents an important addition to the existing literature in terms of the complex geometries that can be designed and analyzed and at the same time constitutes an important refinement over previously established work. In fact, the present work permits the design and analysis of a fully-coupled magnetoelectric plate or sandwich plate with any desired arrangement of the actuators/reinforcements.

1. Introduction

The incorporation of composites, smart composites and nanocomposites into new engineering applications can be facilitated if their properties and behavior can be simulated at the design stage. A priori determination of the macroscopic or effective properties of such structures, for example elastic, thermal, optical and other coefficients is rendered inherently difficult due to such factors as the coupled behavior of drastically different constituents (sensors, actuators, reinforcements, surrounding matrix etc.), the spatial distribution and geometrical characteristics of these constituents, the nature of different loading configurations and conditions and many others. Furthermore, and pertaining to periodic composite structures, another major source of difficulty is related to the fact that the differential equations characterizing their micromechanical behavior are characterized by rapidly varying coefficients. The high frequency of variation of these coefficients is tantamount to the microscopic (or nanoscopic) scale describing the typical dimensions of the periodicity unit of the structures; at the

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same time, and superimposed on this microscopic scale is the macroscopic scale or “slow” scale which is a manifestation of the global formulation of the problem under investigation. The presence of these two superimposed scales, in addition to the aforementioned other sources of difficulty, renders an analytic solution virtually impossible except in cases of simple geometries. Ideally, one is looking for a micromechanical model that is comprehensive enough to capture the important underlying physics of the composite but at the same time not too complicated to be used efficiently and expediently. The best case scenario would involve the model culminating in closed-form expressions for the effective coefficients that can be easily incorporated into analysis and design. One method that can be used to accomplish all these objectives is that of asymptotic homogenization. The mathematical framework of this technique can be found in Bensoussan et al. [1], Sanchez-Palencia [2], Bakhvalov and Panasenko [3], and Cioranescu and Donato [4]. The premise of asymptotic homogenization is based on decoupling the microscopic and macroscopic scales so that each is treated separately. A large class of problems in the fields of elasticity, thermoelasticity, piezoelasticity and others has been solved using asymptotic homogenization. We mention characteristically the comprehensive works of Kalamkarov [5] and Kalamkarov and Kolpakov [6] that provide solutions to many problems in the analysis of the effective and local properties of composite structures, as well as to problems of their design and optimization on account of strength, stiffness and weight minimization. Hassan et al. [7] and Kalamkarov et al. [8] developed micromechanical models for the analysis of, respectively, general 3D network-reinforced composites and thin smart composite shells consisting of a soft matrix reinforced by arbitrarily oriented cylindrical bars. As an interesting special case of the model developed in Kalamkarov et al. [8], the authors determined the stiffness and shear moduli of single-walled carbon nanotubes whereby the covalent bonds between carbon atoms were modeled as cylindrical reinforcements embedded in a matrix of zero rigidity. Hadjiloizi et al. [9,10] developed general 3D models pertaining to piezo-magneto-thermo-elastic composites. Two modeling approaches were examined; one involved the use of the time-varying form of Maxwell's equations as well as dynamic force and thermal balance while the other used the quasi-static approximation of Maxwell's equations. The models were used to analyze thick laminates and 3D grid reinforced composites. Hadjiloizi et al. [11,12] developed a comprehensive micromechanical model for the analysis and design of thin smart composite plates with rapidly varying thickness and used the model to examine rib-reinforced and wafer-reinforced magnetoelectric plates. Other works can be found in Duvaut [13], Duvaut and Metellus [14], Andrianov and Manevich [15], Andrianov et al. [16], Devries et al. [17], Guedes and Kikuchi [18], Yi and Youn [19], Aboudi [20], Bravo-Castillero et al. [21] and many others.

Composites structures made up of piezoelectric and piezomagnetic materials have attracted particular attention in recent years due to the interesting properties they exhibit that make them particularly suited to a broad range of applications; for example their sensitivity to electric, magnetic and thermal fields can be exploited for frequency tunable devices such as resonators and phase shifters, energy harvesting devices, data storage devices and spintronics, biomedical sensors and many other applications, see for example Bichurin et al. [22,23], Bhatra et al. [24], Ju et al. [25], Oh et al. [26], Semenov et al. [27], Fusil et al. [28], Zhai et al. [29], Shen et al. [30] and many others. The properties that make piezoelectric-piezomagnetic composites attractive for such applications are often referred to as product properties and describe the coupling between mechanical, electrical, magnetic and thermal fields, see Nan et al. [31], Srinivasan [32], Kondaiah et al. [33]. Consider for example a piezoelectric-piezomagnetic composite. Applying a magnetic field to this structure will generate a mechanical strain in the piezomagnetic constituent. Provided that there is a satisfactory degree of bonding between the two constituents, the magnetic-field induced strain will be transferred to the piezoelectric phase and in turn induces an electric displacement. Thus, as far as the macroscopic composite is concerned, a magnetic field induces an electric field. The converse is also true and an applied electric field induces a magnetic field. The coefficients which describe this coupling behavior are called magnetoelectric coefficients. It is apparent that magnetoelectric behavior is a characteristic of the overall composite and is not usually exhibited by the individual phases. This also applies to other product properties as well such as pyroelectricity and pyromagnetism which describe the coupling between thermal and electrical fields and thermal and magnetic fields respectively.

Increased interest in magnetoelectricity and other product properties spurred a corresponding upturn in research activities related to their micromechanical modeling. Naturally, both numerical approaches as well as analytic/semi-analytic approaches were employed to examine magnetoelectric structures. We mention characteristically the works of Nan [34], Gopinathan et al. [35], Chopra [36] who gives a detailed account of a number of modeling techniques (analytic, semi-analytic and numerical), Li [37] and others. In general, most research efforts were directed towards laminated structures due to the ease of fabrication of such structures and because more complex geometries would naturally present more difficulties. Harshe et al. [38] and Avellaneda and Harshe [39] obtained the magnetoelectric coefficients of 2-2 piezoelectric/magnetostrictive multilayers for different boundary conditions; through the computation of a figure of merit related to magnetoelectric coupling and the pertinent optimization in terms of constitutive materials and related volume fractions, the authors illustrated the usefulness of such structures in magnetoelectric transducer applications. Li and Dunn [40] elaborated a micromechanical model for piezoelectric/piezomagnetic two-phase composites and used it to obtain closed-form expressions for the effective coefficients for both fibrous and laminated composites using the Mori-Tanaka approach [41]. Particularly noteworthy is the work of Aboudi [20] who developed a general micromechanical modeling platform for the analysis of piezo-magneto-thermo-elastic composites. The author divides the unit cell of the structure into subcells and computes the various field variables by volume averaging of the appropriate equations with a simultaneous consideration of boundary conditions and continuity relationships between the subcells. Finally, expressions for the effective coefficients are determined. The model is illustrated via examples of composites consisting of unidirectional piezoelectric fibers embedded in a piezomagnetic phase and bilaminated structures made up of alternating piezoelectric and piezomagnetic phases. The model is in excellent agreement with the Mori-Tanaka approach as well as the method of cells, see Aboudi [42]. Huang et al. [43] studied piezoelectric/piezomagnetic laminates subjected to coupled bending and stretching loads and concluded that the exhibited degree of magnetoelectric coupling was higher than in the case of pure stretching. Bichurin et al. [44] calculated the magnetoelectric coefficients of piezoelectric/magnetostrictive laminates for various orientations of electric and magnetic fields. This work was later extended to magnetoelectric nanocomposites, see Bichurin et al. [45]. Dong et al. [46] examined the potential of piezoelectric/magnetostrictive laminates in power transformer applications. Among others, the authors examined the effect of the relative thickness of the constituents on the degree of magnetoelectric coupling. Ni et al. [47] examined the magnetoelectric effect in 3-ply polycrystalline multiferroic laminates consisting of a piezoelectric lamina between two ferromagnetic ones. The authors