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Classical radiation reaction in particle-in-cell simulations

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ABSTRACT

Under the presence of ultra high intensity lasers or other intense electromagnetic fields the motion of particles in the ultrarelativistic regime can be severely affected by radiation reaction. The standard particle-in-cell (PIC) algorithms do not include radiation reaction effects. Even though this is a well known mechanism, there is not yet a definite algorithm nor a standard technique to include radiation reaction in PIC codes. We have compared several models for the calculation of the radiation reaction force, with the goal of implementing an algorithm for classical radiation reaction in the Osiris framework, a state-of-the-art PIC code. The results of the different models are compared with standard analytical results, and the relevance/advantages of each model are discussed. Numerical issues relevant to PIC codes such as resolution requirements, application of radiation reaction to macro particles and computational cost are also addressed. For parameters of interest where the classical description of the electron motion is applicable, all the models considered are shown to give comparable results. The Landau and Lifshitz reduced model is chosen for implementation as one of the candidates with the minimal overhead and no additional memory requirements.

1. Introduction

The next generation of high-power lasers is going to reach intensities that will open new doors for exploring a wide range of physical problems with an even wider range of applications. The ELI project [1] is expected to reach laser intensities several orders of magnitude higher than those available today. At intensities $I \sim 10^{23}$–10^{24} W/cm^2 one can expect electron–positron pair production [2–5]. In astrophysics these intensities are relevant for the study of pulsars, blazars, and gamma-ray bursts [6]. High intensity laser–matter interactions can also produce proton and heavy ion beams [7–10] that are of great significance for many applications, the most important being cancer treatment.

At high intensities particle acceleration can be severely limited by the radiation reaction associated with the energy loss via radiation emission [11]. This is important whenever the radiated energy is comparable to the total particle energy. The threshold electromagnetic field intensities to drive these effects vary in the available literature, but they are usually in the range 10^{22}–10^{25} W/cm^2. Many authors have discussed the classical radiation reaction in pursuit of a proper analytical description [12–21] or experimental signatures [22–32]. There is also a rising interest in the effect of the radiation reaction on particle dynamics in astrophysical phenomena [33–36]. In order to perform reliable particle-in-cell (PIC) simulations in the classical radiation-dominated regime, radiation reaction (RR) must be included in the equations of motion for the particles. Though several models for classical radiation reaction have been proposed in the literature, there is not a definite standard choice to apply in PIC codes. In this paper we compare several different models in order to find the most appropriate one for implementation in OSIRIS [37,38]. The chosen model should capture all the relevant physics in the scenarios we are aiming to explore at the lowest possible cost in performance. We test each of them with well studied examples of particle motion in electromagnetic fields where the trajectory can be analytically expressed and estimates for the radiated power/energy can be obtained. Our analysis shows that even though there are conceptual differences between the models considered, in the parameters of interest that could be tested with near-future laser technology all the models give the same description of particle motion. To observe differences, one would need to impose a linear acceleration so high that an extreme electric field required to produce it would render a classical description of an electron trajectory inapplicable. The choice can then be made on the basis of the computational overhead and with this aim the additional computational cost that the implementation of

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each model introduces is presented. Among the models with the lowest computational requirements, we opted to introduce Landau and Lifshitz reduced model in OSIRIS framework. Specific questions associated with the interpretation of classical RR in PIC simulations are also addressed.

This paper is structured as follows. In Section 2 we introduce the radiation reaction models and deal with macro particle interpretation. In Section 3, the behaviour of all the models is investigated in the standard cases of synchrotron radiation and bremsstrahlung. Section 4 underlines the difference between the results with and without radiation reaction for an electron in a laser pulse field and gives an estimate of the threshold for the detectable radiation reaction. The issue of the optimal temporal resolution is addressed in Section 5, and Section 6 contains estimates for the computational overhead for each model. Finally, in Section 7 we state the conclusions.

2. Radiation reaction models

The charged particle motion with radiation reaction is expressed by the Lorentz–Abraham–Dirac (LAD) equation [39]:

$$\frac{dp_\mu}{d\tau} = F_\mu^{EXT} + F_\mu^{RR} \quad \text{where}$$
$$F_\mu^{RR} = \frac{2e^2}{3mc^3}\left(\frac{d^2p_\mu}{d\tau^2} + \frac{p_\mu}{m^2c^2}\left(\frac{dp_\nu}{d\tau}\frac{dp^\nu}{d\tau}\right)\right). \tag{1}$$

Here, F_μ denotes the electromagnetic force four-vector, p_μ is the particle momentum four-vector, e, m are the elementary charge and particle mass respectively, and c is the speed of light. Eq. (1) is derived for a point charge. Unphysical solutions appear, for example, when $F_\mu^{ext} = 0$, where in addition to the solution with a constant velocity, Eq. (1) has a solution where the particle accelerates infinitely (the so-called "runaway solution"). The principle of causality is also violated here with pre-acceleration solutions — these solutions anticipate the change of the force, so the particle accelerates before the force has been applied. The detailed explanation of these problems and suggestions for possible improvements are given in [40,41].

However, even if these problems would be solved, the LAD equation is inconvenient for numerical integration. It is possible to integrate it backwards in time [42], but this is not applicable to many-body problems [43]. Therefore, the LAD equation, expressed by (1), is inadequate for use in a simulation code.

Numerous approximate models have been explored to eliminate the above-mentioned problems and to obtain a more efficient computational approach. We consider a subset of these models appropriate for PIC implementation: B08 [2], LL [44] (used also in [7] and [6]), S09 [45], H08 [46], and F93 [47]. We also consider the LL reduced (LLR) model [48] where the spatio-temporal derivatives of the fields are discarded in the equation of motion because they are shown to have smaller contribution than the particle spin, that becomes important only in the quantum regime. Model B08 keeps only the leading-order term of the LL equation. The H08 model estimates the total energy radiated via the Larmour formula, and then this energy is discounted from the particle in the end of the timestep. All the models apply to the case of relativistic motion that is required to have an appreciable energy loss due to radiation emission. Their validity is limited to the classical domain, where the particle trajectory can still be considered to be a smooth function of time (the individual emission events do not take a great fraction of the particle energy, or in other words, the emitted energy in the Lorentz frame momentarily co-moving with the emitting particle is small compared with mc^2). Models LL and S09 also appear in Ref. [49] that compares the asymptotic solution for equations of motion coming from strong field quantum electrodynamics with several classical equations of motion; their findings show that LL model is consistent with the asymptotic strong field QED description. Therefore, we will identify where the results obtained with other models are the same (or close enough) as with LL within the limits of the validity of the classical description.

To facilitate the analysis for the PIC implementation, we will use the 3-vector form of the equations throughout. The total change of momentum in time depends on the Lorentz force ($\mathbf{F}_L$) and the radiation reaction force ($\mathbf{F}_{RR}$):

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}_L + \mathbf{F}_{RR} \tag{2}$$

where $\mathbf{F}_L$, in CGS units, is given by:

$$\mathbf{F}_L = e\left(\mathbf{E} + \frac{\mathbf{p}}{\gamma mc} \times \mathbf{B}\right). \tag{3}$$

The radiation reaction force ($\mathbf{F}_{RR}$) for all the considered models, in CGS units, is presented in Table 1. Here the radiation back reaction is explicitly given as an additional force acting on the particle, expressed as a function of the electromagnetic fields $\mathbf{E}$, $\mathbf{B}$, the particle momentum $\mathbf{p}$, charge e, mass m and relativistic factor γ, the speed of light c and time t.

Particle-in-cell codes usually employ normalised units to bring all the quantities to similar orders of magnitude and express the physics as a function of fundamental plasma parameters. The normalisation in OSIRIS is as follows: $t \to t\omega_n$, $\mathbf{x} \to \mathbf{x}\omega_n/c$, $\mathbf{p} \to \mathbf{p}/mc = \gamma\mathbf{v}/c$, $\mathbf{E} \to e\mathbf{E}/mc\omega_n$, $\mathbf{B} \to e\mathbf{B}/mc\omega_n$. Here, $\mathbf{x}$ represents a vector in coordinate space, while ω_n is a chosen reference value for the normalising frequency, which, for instance, can be equal to the electron plasma frequency, or the frequency of the laser. In normalised units, the equations of motion of the particles are free of physical constants, except for a dimensionless coefficient

$$k = \frac{2\omega_n e^2}{3mc^3} \tag{10}$$

which appears in the radiation reaction force term. Therefore, when including the radiation reaction force, the particle motion is no longer dependent only on the charge-to-mass ratio as in the Lorentz force $\mathbf{F}_L$. This poses an additional challenge for the PIC implementation of classical radiation reaction.

In standard PIC codes, the system is represented using macro-particles. Every macro-particle has the same charge-to-mass ratio as a single particle, and therefore the dynamics of the macro-particles is the same as the dynamics of the original particle species. However, after including the radiation reaction, this no longer holds. If we examine Eqs. (2), (3) and (9), we can see that the ratio between the radiation reaction force and the Lorentz force is proportional to the cube of the charge, and to the reciprocal value of the squared mass

$$\frac{F_{RR}}{F_L} \propto \frac{e^3}{m^2}. \tag{11}$$

Let us consider a macro particle that represents η electrons. The charge of the macro particle is $e_m = \eta e$, and the mass is $m_m = \eta m_e$. For a single particle with the same mass and charge as the macro particle, the radiation reaction would be η times stronger than in the case of a single electron:

$$\frac{F_{RR}}{F_L} \propto \frac{(\eta e)^3}{(\eta m_e)^2} = \eta\frac{e^3}{m_e^2} \tag{12}$$

and the trajectory of such particle would be different than the trajectory of a single electron (Fig. 1). This result would be equivalent to assuming that η electrons are radiating coherently. As a consequence, the results of a PIC simulation would be