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A physically based model for kink-band growth and longitudinal crushing of composites under 3D stress states accounting for friction

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ABSTRACT

A material model to predict kink-band formation and growth under a 3D stress state is proposed. 3D kinking theory is used in combination with a physically based constitutive law of the material in the kink-band, accounting for friction on the microcracks of the damaged material. In contrast to existing models, the same constitutive formulation is used for fibre kinking and for the longitudinal shear and transverse responses, thereby simplifying the material identification process. The full collapse response as well as a crush stress can be predicted. The model is compared with an analytical model, a micro-mechanical finite element analysis and crushing tests. In all cases the present model predicts well the different stages of kink-band formation and crushing.

1. Introduction

Understanding, modelling and finally predicting the compressive response of continuous Carbon Fibre Reinforced Plastics (CFRP) materials is a long standing and unresolved issue. It is however an important one as kink-band formation is a failure mode responsible for high energy absorption in compression. Currently available meso-models use linear degradation of the stresses based on toughness values for kink band formation [1], which are hard to measure and very scattered. The present model eliminates the need for fibre kinking toughness measurement, and considers the energy absorbed by friction once the material has been fully damaged. Such an approach is important in areas where it is not only necessary to accurately predict the compressive strength (peak stress) but also the post-peak behaviour. For example, the model is interesting for simulation of crash structures in car applications, for predicting the response of bolted joints and for the residual response of composite structures subjected to impact.

Compressive failure of fibre composites may involve splitting failure, kinking failure or a combination of both [2]. Splitting failure often occurs at the fiber-matrix interface within the kink-band as shown in Ref. [3]. Because of the large strains involved, a significant amount of the stored energy can be released during splitting [4]. Thinner fibres and higher fibre volume fractions were shown to promote kinking failure [2]. The current paper is focused on structural carbon fibre composites, which typically fail in kinking for fibre volume fractions between 10 and 60%.

Improving from the first strength models based on kinking induced by microbuckling developed by Rosen [5], the matrix shear strength was later on accounted for by Argon [6] and Budiansky [7]. The resulting equation, Eq. (1), predicts the longitudinal compressive strength, X_c, under the assumption that kink-band formation occurs when the shear strength in the kink-band is reached due to further rotation of initially misaligned fibres

$$X_c = \frac{S_L}{\theta_i + \gamma_c} \tag{1}$$

where S_L is the in-plane shear strength, γ_c the shear strain at failure and θ_i is the initial fibre misalignment. Eq. (1) is a key equation in fibre kinking theory, a framework which has been extended to predict the collapse response [8]. The collapse, or post peak, response was predicted for 2D stress states and using a nonlinear elastic response (Ramberg-Osgood model) for the material in the kink-band [8] or Schapery theory in Ref. [9]. A typical response shows a strain softening after the peak load and a flattening of the curve at larger strains.

Similar collapse responses were predicted by analytical micro-mechanical models based on couple stress theory [10–12]. These models account for fibre bending and show that it can be neglected

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in predicting failure although it plays an essential role in the calculation of kink-band width. The effect of multiaxial 2D loading was later considered in Ref. [13], where it was shown how the axial compressive strength was increased by transverse compression, and reduced by transverse tension.

While some of the previous models have been implemented in a Finite element (FE) framework, they do not handle 3D stress states. Under such stress states, Pinho et al. [1,14] proposed a model based on fibre kinking but where failure initiation is predicted by a Mohr-Coulomb criterion.

In the present contribution, the 3D fibre kinking theory proposed in Ref. [1] is extended to also predict the collapse response. The post peak response follows the work by Refs. [8,9] but provides an improved response of the material in the kink-band compared to these models. To this end, the model uses a constitutive law based on progressive damage enhanced to account for contact and friction at microcrack closure under compressive loading [15]. The work presented here together with the model developed in Ref. [15] provides a unified and physical approach to model crushing both for compressive matrix failure and fibre kinking failure. In particular, the model proposes a physical solution to the different stages of energy dissipation, first during damage formation and second by friction during the crushing.

2. Formulation of the model

The objective of the model is to predict the longitudinal compressive response ($\sigma_{11} < 0$) for a transversely isotropic ply under an arbitrary loading. In the longitudinal direction, loading is strain driven due to the collapse nature of the response and the resulting strain softening. For direct and shear loading in the other directions both stress and strain can be prescribed.

Matrix damage introduced by in-plane shear and the transverse loading is accounted for by using the nonlinear constitutive law derived in Ref. [15]. This constitutive law is also used to describe the response of the material in the kink-band. In short, the fibre kinking response is found from solving (i) a stress equilibrium between applied global stresses and nonlinear local stresses resulting from (ii) the nonlinear constitutive law of the material in the kink-band. The actual rotation of the fibres in the kink-band, θ, is defined as a state variable and resolved from (iii) the strain compatibility. The three conditions (i) to (iii) must be solved simultaneously. This is described below after a description of the conventions.

2.1. Geometrical description and notations

Throughout the manuscript, the conventions and notations defined in Fig. 1 will be used.

Fig. 1(a) shows a micrograph of a kink-band formation with typical parameters, where w is the kink-band width and β is the kink-band angle. A unidirectional lamina under a general compressive stress state is shown in Fig. 1(b). Two coordinate systems are introduced. The first one corresponding to the orientation of the kink-band through the thickness is denoted r and is characterised by the angle ψ. The second (misaligned) coordinate system, m, is associated with the rotating fibres and is characterised by the angle θ, Fig. 1(c). Fig. 1(d) defines the initial and current configurations and associated fibre rotations. The numerical subscripts (11, 22, 12) refer to the global coordinate system and the subscripts using m (11m, 22m, 12m) refer to the misaligned frame, m, also referred to as local frame later on.

2.2. Stress equilibrium

A kink-band forms when nonlinearities occur in the matrix material between fibres rotating under a compressive load. To predict initiation and growth of kink-bands, the equilibrium equations between the applied external stresses and the local stresses need to be solved. In a 3D framework it is assumed that fibres will form planes of kinked fibres, Fig. 1(b), i.e. that no out of plane shear stresses are acting on a kink-band plane. This assumption was introduced in Ref. [1] and gives the following equation for the angle ψ:

$$\psi = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{23}}{\sigma_{22}-\sigma_{33}}\right) \tag{2}$$

Knowing ψ, the stress in the r coordinate system, $\boldsymbol{\sigma}_r$, can be found from the global stress as follows:

$$\boldsymbol{\sigma}_r = \mathbf{T}_\psi \boldsymbol{\sigma} \mathbf{T}_\psi^T \tag{3}$$

where $\mathbf{T}_\psi$ is the transformation matrix for the angle ψ and where $\mathbf{T}_\psi^T\mathbf{T}_\psi = \mathbf{I}$.

The final equilibrium to be solved is expressed in the m coordinate system as

$$\boldsymbol{\sigma}_m - \mathbf{T}_\theta \boldsymbol{\sigma}_r \mathbf{T}_\theta^T = 0 \tag{4}$$

Where $\mathbf{T}_\theta$ is the transformation matrix for the angle θ and where $\mathbf{T}_\theta^T\mathbf{T}_\theta = \mathbf{I}$ and $\boldsymbol{\sigma}_m$ are the local stresses which results from the local strains through the constitutive law defined in the next section $\boldsymbol{\sigma}_m = \mathcal{C}(\boldsymbol{\varepsilon}_m)$.

In the (1_m, 2_m) plane there are only three non-trivial equations

$$\sigma_{11m} - \left(\sigma_{11r}c^2 + \sigma_{22r}s^2 + 2\tau_{12r}sc\right) = 0 \tag{5}$$

$$\sigma_{22m} - \left(\sigma_{11r}s^2 + \sigma_{22r}c^2 - 2\tau_{12r}sc\right) = 0 \tag{6}$$

$$\tau_{12m} - \left(-\sigma_{11r}sc + \sigma_{22r}sc + \tau_{12r}\left(c^2 - s^2\right)\right) = 0 \tag{7}$$

where $c = cos(\theta)$ and $s = sin(\theta)$.The relation between fibre rotation and the in-plane shear in the misaligned frame is established from Fig. 1(d) as

$$\theta = \gamma_{12m} + \theta_i \tag{8}$$

2.3. Longitudinal shear and transverse responses

From Eq. (5)–(7), it can be seen that the longitudinal response derives from the response in the local coordinate system (m). In particular, the longitudinal (in-plane) shear τ_{12m} typically shows a significant nonlinear response, which results eventually in the collapse response in longitudinal compression due to a shear instability [1]. To predict the response and stresses in the kink-band, the constitutive law derived in Ref. [15] is used and is noted $\boldsymbol{\sigma}_m = \mathcal{C}(\boldsymbol{\varepsilon}_m)$. For a given strain state $\boldsymbol{\varepsilon} = (\varepsilon_{11}\ \ \varepsilon_{22}\ \ \varepsilon_{33}\ \ \gamma_{12}\ \ \gamma_{13}\ \ \gamma_{23})^T$, the constitutive law provides the stress state $\boldsymbol{\sigma} = \left(\tilde{\sigma}_{11}\ \ \sigma_{22}\ \ \sigma_{33}\ \ \tau_{12}\ \ \tau_{13}\ \ \tau_{23}\right)^T$, where the longitudinal component is calculated linear elastically, noted with ~. The other components are obtained from a fixed crack model in which the relation between the stresses and strains is obtained from the response in the plane determined at failure initiation and fixed during damage evolution. In this fracture plane, a new set of coordinates (NLT) is defined: normal, longitudinal and transverse to the fibres direction, see Fig. 2. The fixed crack formulation makes it possible to account for physical mechanisms