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Effect of regularized functions on the dynamic response of a clutch system using a high-order algorithm

Youssef Hilali, Bouazza Braikat *, Hassane Lahmam, Noureddine Damil

Laboratoire d'ingénierie et matériaux (LIMAT), Faculté des sciences Ben M'Sik, Université Hassan-II de Casablanca, Sidi Othman, B.P. 7955, Casablanca, Morocco

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ABSTRACT

The main objective of this work is to propose some regularization techniques for modeling contact actions in a clutch system and to solve the obtained nonlinear dynamic problem by a high-order algorithm. This device is modeled by a discrete mechanical system with eleven degrees of freedom. In several works, the discontinuous models of the contact actions are replaced by the smoothed functions using the hyperbolic tangent. We propose, in this work, to replace the discontinuous model by a regularized model with new continuous functions that permit us to search the solution under Taylor series expansion. This regularized model approaches better the discontinuous model than the model based on the smoothing functions, especially in the vicinity of the zone of singularities. To solve the equations of motion of discrete mechanical systems, we propose to use a high-order algorithm combining a time discretization, a change of variable based on the previous time, a homotopy transformation and Taylor series expansion in the continuation process. The results obtained by this modeling are compared with those computed by the Newton–Raphson algorithm.

1. Introduction

In contrast to gears, which are considered to be motion transmission elements by obstacles, the clutch is a complex mechanism commonly used in motors, for transmitting the motion by means of a high friction effect between adjacent disks. The clutch plays an important role, not just in the commitment and the separation between the motor and the gearbox, but also in the prevention of the damage of the box speed and the vibration reduction of the rest of the vehicle when there is a modification in the rotation frequency of the disk. During the past years, several studies [1–7] have been developed to describe the nonlinear dynamic behavior of the clutch system in order to study the effect of different key parameters that affect its movement. In many studies, the clutch is modeled by a discrete mechanical system composed of torsional and linear elastic springs of negligible mass and of damping elements and disks assumed to be rigid.

Walha et al. [7] have studied a defective clutch modeled by a mechanical system with eleven degrees of freedom by introducing nonlinearities of the dry friction, a two-stage stiffness, and a spline clearance in order to analyze the effect of defects in angular misalignment and in parallelism on movement. Recently, Walha et al. [8] have proposed a new two-stage

* Corresponding author.
E-mail address: b.braikat@gmail.com (B. Braikat).

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model involving a helical-gear clutch system with twenty-seven degrees of freedom. The differential equations are solved numerically by the Runge–Kutta integration scheme [9] using smoothing by the hyperbolic tangent function. Thus, they have showed that the eccentricity defect affects the nonlinear dynamic behavior of the studied mechanism.

The assembly of different technical components in the clutch system leads to localized contact nonlinearities and important couplings. These nonlinearities are often modeled by piecewise linear functions. To overcome the difficulty of the discontinuity in the numerical integration, some researchers have used a linearization technique that consists in transforming the nonlinear problem into linear problems. But most authors [3,7,10–13] have used the smoothing procedure for the treatment of contact actions.

Kim et al. [14] have proposed four ways of smoothing discontinuous functions using the hyperbolic tangent and arc-tangent functions that provide a good approximation with respect to the hyperbolic-cosine function and the quintic spline. They have proved it by studying the influence of the smoothing factor on the frequency response of a system that contains a clearance nonlinearity. Works by Driss et al. [6,15] and Walha et al. [7] are based on the use of the hyperbolic tangent function to smooth three types of singular functions of a torsional model. Duan et al. [12] have investigated the dynamics of a mechanical oscillator with a pre-load nonlinearity by the multi-harmonic method using the arc-tangent function for smoothing the non-analytical relationship. Duan et al. [3] have used the hyperbolic tangent function to approximate the classical function of Coulomb friction.

According to Duan et al. [10], the value of the conditioning factor should be carefully selected to ensure the appropriate representation of the discontinuous function. The increase in the value of the smoothing factor improves the approximation of the discontinuous models. However, a larger value of this parameter can lead to numerical instabilities that generate a considerable computation time [3]. In another work [13], it has been shown that, for small values of this parameter, the convergence is fast, but it is not preferred, since it induces a bad approximation and therefore the calculated response may not be sufficiently precise.

In this work, we are interested in the numerical simulation of nonlinear dynamics of a clutch system using a new version of a high-order algorithm [16–20] with a new modeling of the different contact actions. Our numerical algorithm is essentially based on the discrete model proposed in the article [15], which represents the clutch mechanism by a spring-mass-damper system, and whose equations of motion are solved using the numerical integration scheme of Runge–Kutta. In our modeling, we propose a new improved version of the modeling of different contact actions in a clutch mechanism existing in the literature in order to achieve a good approximation, and we try to obtain a realistic response. To show the effectiveness of the numerical approach to simulate the nonlinear dynamic response of the considered mechanical system, we have chosen as a reference the Newton–Raphson algorithm [21] coupled with the Newmark integration scheme with the use of the hyperbolic tangent function, and we used thereafter the regularization developed in this work. The high-order approach is used for the purpose of reducing the computation time. This approach, allowing the construction of the curve solution branch by branch, is based on coupling power series expansion and on a continuation process. In our analysis, we study the effect of different regularization parameters on the nonlinear dynamic responses of the considered clutch system.

2. Dynamic model of the clutch mechanism

The clutch mechanism is modeled by a physical system shown in Fig. 1. This system is comprised of two blocks. The first contains the flywheel and the cover (m_1, I_1), the diaphragm spring and the pressing plate (m_2, I_2), while the second is comprised of the friction shoes (m_3, I_3), the hub of the clutch (m_4, I_4) and the rest of the transmission (m_5, I_5), with I_i and m_i being respectively the mass and the torsional inertia of the ith element of the clutch. Each block is supported by a flexible bearing of bending rigidity k_{z_i} and of traction–compression stiffness k_{x_i} and k_{y_i}. This bearing is connected in parallel with the damping elements c_{x_i}, c_{y_i} and c_{z_i}, where i is the number of blocks $(i = 1, 2)$. The contact between the two blocks is performed by the torques and the friction forces. The friction torque $T_\text{f}(\dot{\delta}_2)$ represents the torsional friction between the pressing plate and friction shoes and $T_\text{f}(\dot{\delta}_3)$ represents the friction torque between the flywheel and the friction shoes. In both cases, the movement of these blocks, in the plan xy, induces tangential friction forces $F_{T_{x_{1/2}}}$ and $F_{T_{y_{1/2}}}$ that express the actions applied by the first block on the second in both directions x and y, respectively. The action of the spring diaphragm, which allows the components of the first block to turn at the same angular speed as those of the second block, is modeled by a torsional spring of stiffness k_d and a negligible mass. The cover applies a pressure P to maintain both the flywheel, pressing plate, and friction shoes in permanent contact to ensure a perfect transmission. The actions of the torsion springs which bind the splined hub with friction shoes are represented by the term $K_\text{dss} f_\text{dss}(\delta_i)$, with K_dss being the equivalent stiffness of the equivalent linear spring and $f_\text{dss}(\delta_i)$ is the contact force. The backlash space in the splined attachment between the hub and the output shaft can be defined by the term $K_\text{sc} f_\text{sc}(\delta_i)$, where K_sc represents the torsional stiffness of the shaft and $f_\text{sc}(\delta_i)$ is the function that models the loss of contact. The equation of motion of this discrete model verified by the vector of generalized coordinates can be written in the following matrix form:

$$\begin{aligned}[M]\{\ddot{q}\} + [C]\{\dot{q}\} &+ [K]\{q\} + f_\text{dss}(\delta_4)\{\varphi_\text{dss}\} + f_\text{sc}(\delta_5)\{\varphi_\text{sc}\} + T_f(\dot{\delta}_2)\{\varphi_2\} \\ &+ T_f(\dot{\delta}_3)\{\varphi_3\} + F_{T_{x_{1/2}}}\{\varphi_{T_x}\} + F_{T_{y_{1/2}}}\{\varphi_{T_y}\} = \{F_\text{ext}(t)\}\end{aligned} \tag{1}$$

where $[M]$ represents the mass matrix given by: