ARTICLE IN PRESS CRAS2B:3500

C. R. Mecanique ••• (••••) •••–•••

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

A century of fluid mechanics: 1870–1970 / Un siècle de mécanique des fluides : 1870–1970

Joseph Boussinesq's legacy in fluid mechanics

Olivier Darrigol

CNRS: UMR SPHere, Université Paris-7, France

ARTICLE INFO

Article history:
Received 17 October 2016
Accepted 17 November 2016
Available online xxxx

Keywords:
Joseph Boussinesq
Solitary waves
KdV equation
Open-channel flow
Boussinesq approximation
Physical mechanics
Adhémar Barré de Saint-Venant

ABSTRACT

Joseph Boussinesq was the most prolific of all French contributors to nineteenth-century fluid mechanics. His scientific production included a novel theory of solitary waves, the KdV equation for finite deformations of the water surface in an open channel, a systematic study of open channel and pipe flow based on the concept of effective viscosity, pioneering derivations of boundary layers and entrance effects, new exact solutions of the Navier–Stokes equation under geometrically simple boundary conditions, and the 'Boussinesq approximation' for heat convection in a moving fluid under gravity. Although his extraordinary skills were quickly recognized and rewarded, other experts in the field were often unaware even of his most important results and they ended up rediscovering some of them. Boussinesq's unusual background and the resulting peculiarities of his style explain this problematic diffusion. They also account for the richness of his legacy.

1. Introduction

Joseph Valentin Boussinesq was born in 1842 in the small town of Saint-André-de-Sangonis in Southern France, to a family of humble cultivators who would have wished him to stay with them. Young Boussinesq enjoyed playing on the Hérault River, throwing stones into its waters, diving deep into it, or just watching its sometimes impetuous flows. Intensely curious about natural phenomena, he questioned the local school teacher about the water waves and whirls he knew how to create. At the age of nineteen, he got his bachelor degree in mathematics at Montpellier and went on to teach this subject in provincial colleges for a few years. During this period, he read the French classics of mathematical physics and tried his hand on difficult theoretical problems. His teaching did not go as well as his reading. He had a weak voice and little authority over classes of boisterous teenagers. In 1864, in the small college of Le Vigan, an inspector was surprised to find, on Boussinesq's desk, a copiously annotated volume of Laplace's *Mécanique céleste*. This incident won Boussinesq a promotion to a more important college in Gap in the Alps. There he had enough time to begin a dissertation on the theory of optics (see [1,2]).

Boussinesq's first publication, a brief note of 1865 in the *Comptes rendus* [B1], bore on elasticity theory applied to the optical ether and was read by the great elasticity theorist Gabriel Lamé. His colleague Émile Verdet, who then was the most competent French expert on optics and molecular theories, directed a dissertation in which Boussinesq sought a complete explanation of the interplay of ether and matter in various phenomena including optical dispersion, crystal optics, optical rotation, and the optics of moving bodies. The outcome was a remarkably powerful theory that anticipated the central features of Hendrik Lorentz's later electromagnetic theory (see [3], pp. 256–258). This theory appeared with some delay in 1868 and not as a doctoral thesis [B5, B6]. In place of it, Boussinesq successfully defended a thesis on the propagation

E-mail address: darrigol@paris7.jussieu.fr.

http://dx.doi.org/10.1016/j.crme.2017.05.008

of heat in anisotropic media at the end of 1867 [B3]. The reason for this change of topic was Verdet's untimely death in June 1866 and the Sorbonne's choice for examining the dissertation: Charles Briot. This disciple of Augustin Cauchy had recently developed an optical theory in which the molecules of matter modified the density of the surrounding ether, whereas Boussinesq assumed the interstitial ether to be completely identical with the ether in vacuum. In order to avoid confrontation, Boussinesq replaced his optical work with a memoir he had recently written on heat propagation [see B32, pp. 267n–268n].

In the same period, Boussinesq addressed two other questions: the general form of molecular forces [B2], and viscous flow in narrow tubes [B4]. He got the attention of the engineer, physicist, and freshly elected Academician Adhémar Barré de Saint-Venant, who became his friend, spiritual guide, and zealous supporter. One may wonder what brought Boussinesq to investigate, at nearly the same time, questions of optics, elasticity theory, molecular theory, and fluid mechanics. One may also wonder what brought Boussinesq and Saint-Venant so close together. The answers to these two questions are intimately related.

Saint-Venant regarded himself as the continuator of a tradition inaugurated by Laplace, Siméon Denis Poisson, Augustin Cauchy, Augustin Fresnel, and André Marie Ampère, in which physical bodies were regarded as collections of point-like molecules interacting through distance forces. The elasticity of solid bodies, the rigidity of the optical ether, the compressibility of gases, the viscosity of fluids, and thermal phenomena were all regarded as resulting from this molecular structure. In Saint-Venant's philosophy, the first task of mathematical physics was to define a clear molecular picture from which, by averaging over the many molecules of macroscopic elements, differential equations could be derived. The molecular foundation could then be largely ignored and approximation procedures were devised to solve concrete problems of practical interest. This molecular-based and step-wise physics, which Poisson called *mécanique physique*, largely dominated French mathematical physics in the first third of the nineteenth century; but it had much receded by 1850. One reason was the demise of two central concepts of Laplacian physics: the imponderable, molecular fluids of heat and light. Another reason was the empiricist promotion of theories more directly related to observation. A third and last reason was the preference of the mechanics section of the Paris Academy for the *mécanique rationnelle* of ideal rigid bodies, inextensible threads, and incompressible fluids. Physical mechanics nonetheless survived this hostile environment thanks to the few adepts of Cauchy's molecular optics and to the sustained efforts of Saint-Venant and Boussinesq, who both believed that molecular mechanics remained the best foundation for all pure and applied physics and both ignored domains of physics, such as electromagnetism, that did not fit in this theoretical frame (see [4]; [5], pp. 233–234; [6], chap. 5).

In his very first works on optics and elasticity theory, Boussinesq adopted a molecular foundation from which he promptly derived macroscopic partial differential equations. The skills required in solving these equations being nearly the same as those needed for the equations of fluid mechanics, he was naturally driven to fluid-mechanical problems. He excelled in finding new analytical solutions where others would not even try, for instance for the Poiseuille flow in a tube of triangular section [B7]. As we will later see, he also had a unique flair in finding approximate equations of practical interest. On the conceptual side, he sought a molecular understanding of the distinction between solid and liquid. This is why, in 1867 [B2], he propounded a general molecular force formula in which the force depended not only on the distance between the two interacting molecules (as Saint-Venant had assumed), but also on the density of the surrounding molecules. In general, Boussinesq believed in a molecular foundation of mechanics and physics, and he developed his own vibrational theory of heat in an Amperean tradition distinct from the British and German kinetic theory of gases (see [B25]; [6], chap. 5; [7,8]).

The affinities between Saint-Venant and Boussinesq should by now be evident: they both pursued a three-stage physics involving molecular foundations, partial differential equations, and clever approximation strategies. They both favored highly mathematical theories that did not lose sight of practical engineering interests, and they occasionally injected empirical input into the theory. It is difficult to decide to which extent Saint-Venant determined the preferences of his protégé. What is certain is that from 1868 he frequently reported on Boussinesq's memoirs for the Academy of Sciences, that he suggested improvements and new problems, and that both men exchanged many letters on problems, methods, philosophy, and religion.[1] Plausibly it was Saint-Venant who directed Boussinesq from the Poiseuille flow to problems of hydraulic interests.

Three years after his doctoral thesis, in 1873, Boussinesq won a physics chair at Lille University. In 1886 he obtained the more prestigious chair of mechanics at the Sorbonne. He entered the Academy of Sciences in the same year, a few days after his mentor and ardent supporter Saint-Venant passed away. In 1896 he succeeded Henri Poincaré on the chair of mathematical physics and probability at the Sorbonne. He retired in 1918 and died in 1929.

A large proportion of Boussinesq's works were devoted to fluid mechanics and hydraulics. According to one of his disciples, Auguste Boulanger, he wrote no less than 1800 pages on this topic. It is dubious that anyone read the totality of this corpus. So it remains possible that important results of Boussinesq's have been overlooked to this day. Until recently, it was for instance not known that he had obtained the KdV equation some twenty years before Korteweg and de Vries. The best one can do with a limited amount of time and patience is to identify a few major results and situate them in Boussinesq's scientific biography.

[1] The correspondence between Boussinesq and Saint-Venant is held at the library of the Institut de France.

Please cite this article in press as: O. Darrigol, Joseph Boussinesq's legacy in fluid mechanics, C. R. Mecanique (2017), http://dx.doi.org/10.1016/j.crme.2017.05.008