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Invariant solutions in a channel flow using a minimal restricted nonlinear model

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Simulations using a Restricted Nonlinear (RNL) system, where mean flow distortion resulting from Reynolds stress feedback regenerates rolls, is applied in a channel flow under subcritical conditions. This quasi-linear restriction of the dynamics is used to study invariant solutions located in the bulk of the flow found recently by Rawat et al. (2016) [14]. It is shown that the RNL system truncated to a single streamwise mode for the perturbation supports invariant solutions that are found to bifurcate from a relative periodic orbit into a travelling wave solution when the spanwise size is increasing. In particular, the travelling wave solution exhibits a spanwise localized structure that remains unchanged for large values of the spanwise extent as the invariant solution lying on the lower branch found by Rawat et al. (2016) [14]. In addition, travelling wave solutions provided by this minimal RNL system are self-similar with respect to the Reynolds number based on the centreline velocity, and the half-channel height varying from 2000 to 5000.

1. Introduction

The investigation of relative invariant solutions for wall-bounded flows with homogeneous spatial directions, such as pipes and channels, helped to achieve a considerable step forward in the understanding of phase space trajectories leading to turbulence, under subcritical conditions.

In particular, travelling waves and/or relative periodic orbits have been found at Reynolds numbers much lower than the one corresponding to the onset of an exponential mode (see for instance Duguet et al. [1], Kerswell and Tutty [2], Waleffe [3] and Schneider et al. [4] for pipe, channel and Couette flows, respectively). In addition, the linearized dynamics about these solutions are found to be unstable for most of them.

In that context, several efforts have been made to show that coherent structures observed in wall turbulence result from close passes to unstable invariant solutions [5]. For instance, travelling wave solutions of channel flow concentrated near walls were numerically studied by Jimenéz and Simens [6], Gibson and Brand [7]. Previous authors suggested that these travelling waves correspond to elemental version of near-wall coherent structures. In particular, they bear strong similarities

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with simplified flow motions obtained by direct numerical simulations (DNS) using a "minimal" system (i.e. the smallest computational box in which turbulence may be sustained) [8,9]. A relative periodic orbit solution obtained in a minimal channel flow unit is also suggested by Toh and Itano [10] as the fundamental process of near-wall turbulence.

These solutions feature a sinuously bent low-speed streak flanked by a pair of quasi-streamwise vortices that are organized into a self-sustained cycle (see Waleffe and Kim [11]; and Duriez et al. [12] for experimental evidence). Recently, Rawat et al. [13] and Rawat et al. [14] computed invariant solutions in channel flows at Reynolds numbers based on the centreline velocity and the half channel height varying from 2000 to 5000. While invariant solutions described above are concentrated near the walls, the latter occupy the bulk of the flow. These solutions consist also in sinuous streamwise streaks, periodically or continuously forced by quasi-streamwise vortices in a self-sustained process. Rawat et al. [14] have shown that two branches of travelling wave solutions bifurcate from a relative periodic orbit when the spanwise extent is increased. While the lower branch of travelling wave solutions is characterized by a spanwise-localized pattern, structures lying on the upper branch develop multiple streaks when the spanwise size is increased. These solutions are seen to persist in the turbulent regime, giving some evidence that they are linked to self-sustained large-scale coherent motions populating the outer region [15].

Therefore, the important role of these invariant solutions in bypass transition and low-Reynolds-number turbulence requires the development of numerical methods and models aiming to overcome their expensive computational cost. For that purpose, Thomas et al. [16] and Farrell et al. [17] have recently developed a low-order model, referenced as the Restricted Nonlinear model (RNL), which involves key elements of the self-sustained process. In particular, the RNL model relies on a coupled system of equations reproducing the amplification of the streak secondary instability whose quadratic interactions regenerate rolls that induce streaks by lift-up effect and close the loop of the self-sustained process. Furthermore, Thomas et al. [16] show a close correspondence between RNL models and DNS for the case of a turbulent plane Couette flow at low-Reynolds number. In addition, the RNL system is seen to sustain a turbulent state with a small number of streamwise modes.

Hence, the goal of this work is to further investigate RNL models in reproducing invariant solutions found recently by Rawat et al. [14]. Such an analysis may thus provide a promising tool to analyze the emergence of large-scale motions in wall-bounded turbulent flows with relatively low degrees of freedom. This paper is organized as follows. Section 2 contains a derivation of the minimal RNL model where only one streamwise varying mode is considered. Section 3 is devoted to reproduce invariant solutions found by Rawat et al. [14] using the minimal RNL model. We will explore the evolution of these invariant solutions when the Reynolds number is varying from 2000 to 5000 for a wide range of spanwise sizes. Finally, a discussion follows in section 4.

2. Governing equations and numerical methods

In this section, we describe briefly the system of equations that is used. The latter is based on the previous work of Waleffe and Kim [11] and has also been recently considered by Thomas et al. [16] in the study of the self-sustaining process that maintains turbulence in Couette flow. Accounting that roll/streak dynamics is a key element of invariant solutions in wall-bounded flows, we will consider hereafter a model reproducing the so-called self-sustained process, which states that streaks are generated by the superposition of streamwise rolls and a shear flow (through the lift-up effect), being in turn sustained by the continual reformation of the roll resulting from nonlinear interactions of the streak instability mode.

Following Farrell et al. [17], we consider a plane channel flow where a constant mass flux is maintained through a time-varying pressure gradient referenced as $G(t)$. The streamwise, wall-normal and spanwise coordinates are x, y, and z, respectively. The lengths of the channel in x, y and z are L_x, $2h$ (where h is the half-channel height) and L_z, respectively. We introduce a streamwise average operator: $[\bullet]$. The instantaneous velocity field, $\mathbf{u} = (u, v, w)^{\mathrm{t}}$ (where u, v and w are the streamwise, wall normal and spanwise velocity components, respectively) is decomposed into its streamwise mean value ($\mathbf{U} = (U, V, W)^{\mathrm{t}} = [\mathbf{u}]$) and a perturbation, $\mathbf{u}' = (u', v', w')^{\mathrm{t}}$, such as $\mathbf{u} = \mathbf{U} + \mathbf{u}'$. The pressure is similarly written as: $p = -G(t)x + P + p'$. The Navier–Stokes equations for an incompressible flow are then rewritten into a coupled system for the streamwise mean velocity field and the perturbation. The set of equations for the streamwise mean velocities is given below:

$$\frac{\partial U}{\partial t} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} = G(x) + \nu\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) - \frac{\partial [uv]}{\partial y} - \frac{\partial [uw]}{\partial z} \tag{1a}$$

$$\frac{\partial V}{\partial t} = -\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right) - \frac{\partial [vv]}{\partial y} - \frac{\partial [vw]}{\partial z} \tag{1b}$$

$$\frac{\partial W}{\partial t} = -\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2}\right) - \frac{\partial [wv]}{\partial y} - \frac{\partial [ww]}{\partial z} \tag{1c}$$

$$\frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0, \tag{1d}$$

with ν is the kinematic viscosity. The roll/roll interactions are assumed to be high-order terms [11] and are neglected in equations (1). The perturbation field is governed by: