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On the developments of Darcy's law to include inertial and slip effects

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ABSTRACT

The empirical Darcy law describing flow in porous media, whose convincing theoretical justification was proposed almost 130 years after its original publication in 1856, has however been extended to account for particular flow conditions. This article reviews historical developments aimed at including inertial and slip effects (respectively, when the Reynolds and Knudsen numbers are not exceedingly small compared to unity). Despite the early empirical extensions to include inertia and slip effects, it is striking to observe that clear formal derivations of physical models to account for these effects were reported only recently.

1. Introduction

Flows in porous media are of interest in numerous applications ranging from hydrology, hydrocarbon recovery, gas and nuclear waste storage, to drying of wood, transfer in food products or in living tissues to cite but a few. The main characteristic of this particular domain of fluid mechanics lies in the (sometimes extreme) complexity of the geometry of the channels where the flow takes place. Additionally, in many situations, this geometry is unknown in its very details and may vary over more or less long distances characteristics of heterogeneities. Within this context, the physical description of the flow in such materials may appear to be a tremendous challenge.[1] This certainly explains why empiricism remained so strong and lasted longer than in many other fields of fluid mechanics. In many situations, the interest is not in the details of the flow within the pores but rather in the flux-to-force governing laws at length scales including a large numbers of pores, although comprehensive analyses at the pore scale remain the corner stone in any progress towards the derivation of governing laws at larger scales. Clearly, active research in the description of transfer in porous materials was triggered by the publication of Darcy's law in the middle of the 19th century and the emergence of a key macroscopic physical characteristic of a porous medium, namely the ability of a fluid to flow through it, *i.e.* its permeability.

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[1] The one-phase slow flow is probably one of the simplest mechanism one can think about and there is a lot of other tremendously more complex physical processes in porous media of relevance from both scientific and industrial points of view, including multiphase flows, compressible flows, phase change, deformable porous media, reactions in multicomponent systems, etc.

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1.1. Darcy's law as an empirical basis

Ever since its empirical formulation in 1856, Darcy's law [1] has been a hallmark in modeling momentum transport through porous media. In this classical publication, there is a section entitled *Détermination des lois d'écoulement de l'eau à travers le sable*, dedicated to the study of water flows through a bed of sand where the following relation is proposed (see page 594 in [1]):

$$q = k\frac{s}{e}(h+e) \tag{1}$$

where q is the volumetric flow rate, s is the cross section of the sand bed, e is the bed width, h is the pressure (or head) difference between the surface and the base of the sand bed, and finally k was proposed as a coefficient that depends on the permeability of the bed[2] and on the properties of the fluid. For an excellent review about the origin of Darcy's law, the interested reader is referred to the work by Zerner [3]. The use of this simple relation requires that the only resistance to the flow through the porous medium is due to viscous stresses induced by an isothermal, creeping (or laminar) steady flow of a Newtonian fluid within an inert, rigid and homogeneous porous medium. However, the lack of a rigorous upscaling technique prevented a formal derivation of this equation until the late 1960s, as it will be detailed later.

For a very long period of time – around fifty years – this law has been essentially verified experimentally in its global form, but was not considered in a local differential form nor derived on a theoretical basis. One finds a differential expression in the analysis of the flows in aquifers by J. Boussinesq [4] as a result of an analogy with heat transfer in a continuum. This work also reports an extension of the flow-rate-to-head-gradient relationship to non-homogeneous media. A formal derivation of a 1D local expression of this law obtained from the solution to the Stokes equation for a flow parallel to a regular array of infinite parallel cylinders (sufficiently apart from each other, *i.e.* for relatively large porosities) is due to Emersleben in 1925 [5]. A derivation mainly based on dimensional analysis was later proposed by Muskat and Botset in 1931 [2] for a compressible flow in which the pressure difference is recognized to be replaced by the difference of the squares of the pressures.

Substantial literature will then appear during the 1950s, in which many different approaches to demonstrate Darcy's law will be tested (see for instance [6] and references therein). Although these articles helped progressing into the understanding of the applicability of Darcy's law, almost all of them relied on analogies, hypotheses or postulates that left them incomplete. The first extension to three-dimensions and to non-isotropic materials was reported by Hall [7], who introduced a *permeability tensor*, which is also based on some pre-requisites (see in particular Eq. (17) therein and the way the permeability is identified).

Despite the lack of formal derivation of Darcy's law, which can be expressed for a 1D flow in the x-direction as [8]

$$q = -\frac{Ks}{\mu}\frac{\partial \langle p_\beta \rangle^\beta}{\partial x} \tag{2}$$

the meaning of the permeability and its relationship to the underlying pore structure focused closed attention. In the above expression, K is the permeability having units of m^2 and μ the fluid viscosity. In addition, $\langle p_\beta \rangle^\beta$ is the intrinsically-averaged pressure in the porous medium, defined as:

$$\langle p_\beta \rangle^\beta = \frac{1}{V_\beta}\int_{\mathscr{V}_\beta} p_\beta \, \mathrm{d}V \tag{3}$$

Here, $\mathscr{V}_\beta$ is the domain (of volume V_β) occupied by the fluid phase β within a representative averaging domain (or REV) (see Fig. 1), and p_β is the pore-scale pressure.

An early estimate of K was inspired by an analysis due to Blake in 1922 [9] of flow over packings of grains of different shapes and a comparison to flows in capillary tubes that resulted in the following estimate

$$K = \frac{1}{k_0 S_0^2}\frac{\varepsilon^3}{(1-\varepsilon)^2} \tag{4}$$

where ε is the porous medium porosity, S_0 denotes the specific surface of the particles and it is defined as the ratio of the area of the particle to its volume. The coefficient S_0, related to the effective particle diameter, d_p, was identified from an analogy with spherical particles by

$$S_0 = \frac{6}{d_p} \tag{5}$$

[2] In [1], H. Darcy indicated that k "*depends on the permeability of the sand layer*". In fact, k is the hydraulic conductivity, having the unit of a length per unit time. The intrinsic permeability as a physical quantity, denoted by K (or $\mathbf{K}$ in tensorial form) in the present article, appeared later in the literature. It seems that M. Muskat (see for instance [2]) was the first who used this coefficient.

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