



Heterogeneous linearly piezoelectric patches bonded on a linearly elastic body

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ABSTRACT

In [1], we studied the response of a thin homogeneous piezoelectric patch perfectly bonded to an elastic body. Here we extend this study to the case of a very thin heterogeneous patch made of a periodic distribution of piezoelectric inclusions embedded in a linearly elastic matrix and perfectly bonded to an elastic body. Through a rigorous mathematical analysis, we show that various asymptotic models arise, depending on the electromechanical loading together with the relative behavior between the thickness of the patch and the size of the piezoelectric inclusions.

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1. Introduction

As the essential technological interest of piezoelectric patches is the monitoring of a deformable body they are bonded to, here, in the same spirit as [1], we intend to propose various asymptotic models for the behavior of the body through the study of the system constituted by a very thin patch made of a periodic distribution of piezoelectric inclusions embedded in a linearly elastic matrix (studied in [2,3]) perfectly bonded to a linearly elastic three-dimensional body.

A reference configuration for the linearly elastic body is a domain Ω of $\mathbb{R}^3$ assimilated to the Euclidean physical space with basis $\{e_1, e_2, e_3\}$. The set Ω lies in $\{x_3 < 0\}$ and a part of its Lipschitz-continuous boundary $\partial\Omega$ is a nonempty domain S of $\{x_3 = 0\}$ such that $S \times (-L, 0)$ is included in Ω for some positive real number L . The patch perfectly bonded to the body occupies $B^h = S \times (0, h)$ and is made of a periodic distribution of linearly piezoelectric inclusions perfectly bonded to a linearly elastic and electrically insulated matrix. More precisely, let $Y = (0, 1)^2$ and Y^* be a subdomain strongly included in Y and $I_\varepsilon = \{i \in \mathbb{Z}^2; \varepsilon(i + Y) \subset S\}$ then if $s = (\varepsilon, h)$ is the pertinent couple of geometrical parameters of the patch, the piezoelectric inclusions occupy $B_1^s = S_1^\varepsilon \times (0, h)$, $S_1^\varepsilon = \cup_{i \in I_\varepsilon} \varepsilon(i + Y^*)$, while the elastic matrix occupies the remaining part of B^h . Let $\mathcal{O}^h := \Omega \cup S \cup B^h$. The body is clamped on a part Γ_0 of $\partial\Omega \setminus S$ with a positive two-dimensional Hausdorff measure $\mathcal{H}_2(\Gamma_0)$ and subjected to body forces and surface forces on $\Gamma_1 := \partial\Omega \setminus (S \cup \Gamma_0)$ of densities f and $\mathcal{F}$. The whole patch is free of mechanical loading and no electric charges lay in the inclusions. We define $S^\delta := S + \delta e_3, \forall \delta \in \mathbb{R}$ and, if $u^s, e(u^s)$ and σ^s denote the fields of displacement, strain and stress in $\mathcal{O}^h$ and φ^s, D^s stand for the electric potential and the electric displacement, then part of the equations describing the electromechanical equilibriums read as:

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$$\begin{cases} \operatorname{div} \sigma^s = \tilde{f} \text{ in } \mathcal{O}^h & u^s = 0 \text{ on } \Gamma_0 & \sigma^s n = \mathcal{F} \text{ on } \Gamma_1 & \sigma^s n = 0 \text{ on } S^h \cup \partial S \times (0, h) \\ \operatorname{div} D^s = 0 \text{ in } B_I^s & D^s n = 0 \text{ on } \partial S_I^s \times (0, h) \\ \sigma^s = a e(u^s) \text{ in } \Omega & (\sigma^s, D^s) = \frac{1}{h} M^\varepsilon(e(u^s), \nabla \varphi^s) \text{ in } B^h \end{cases} \quad (1)$$

$\tilde{f}$ is the extension of f to B^h by 0, n is the unit outward normal, a denotes the elasticity tensor, which satisfies:

$$a \in L^\infty(\Omega; \operatorname{Lin}(\mathbb{S}^3)), \exists c > 0; c|e|^2 \leq a(x)e \cdot e \quad \forall e \in \mathbb{S}^3 \text{ a.e. } x \in \Omega \quad (2)$$

where $\operatorname{Lin}(\mathbb{S}^N)$ is the space of linear operators on the space $\mathbb{S}^N$ of $N \times N$ symmetric matrices where the inner product and the norm are noted $\cdot$ and $|\cdot|$ as for $\mathbb{R}^3$. If $\mathbb{K} := \mathbb{S}^3 \times \mathbb{R}^3$ is equipped with an inner product and a norm also denoted as previously and $\mathcal{X}_{Y^*}$ and $L_\#^\infty(Y; \operatorname{Lin}(\mathbb{K}))$ denoting the characteristic function of Y^* and the space of Y periodic elements of $L^\infty(\mathbb{R}^2; \operatorname{Lin}(\mathbb{K}))$, respectively, then M^ε is defined by:

$$M^\varepsilon(x) = M(\hat{x}/\varepsilon) \quad \forall x \in B^h \quad M \in L_\#^\infty(Y; \operatorname{Lin}(\mathbb{K})) \text{ s.t. } M = \begin{bmatrix} \alpha & -\mathcal{X}_{Y^*}\beta \\ \mathcal{X}_{Y^*}\beta^T & \mathcal{X}_{Y^*}\gamma \end{bmatrix} \quad (3)$$

with

$$\exists \kappa > 0 \text{ s.t. } \kappa |k|^2 \leq M(y)k \cdot k \text{ a.e. } y \in Y, \quad \forall k \in \mathbb{K} \quad (4)$$

and, in all the sequel, for any function w of $L^2(Y^*; \mathbb{R}^N)$, $\mathcal{X}_{Y^*}w$ is the function defined by $\mathcal{X}_{Y^*}w = 0$ in $Y \setminus Y^*$, $\mathcal{X}_{Y^*}w = w$ in Y^* .

The models indexed by $p = (\hat{p}, p_3)$, $\hat{p} = (p_1, p_2) \in \{1, 2\}^2$, $p_3 \in \{1, 2, 3\}$ will be distinguished according to the additional necessary boundary conditions on S_I^ε and $S_I^s := S_I^\varepsilon + h e_3$. The case $p_1 = 1$ corresponds to a condition for the electric displacement on S_I^s ,

$$D^s n = q^s \quad \text{on } S_I^s, \quad (5)_1$$

q^s being a density of charges, while $p_1 = 2$ corresponds to a condition of a given electrical potential:

$$\varphi^s = \varphi_0^s \quad \text{on } S_I^s \quad (5)_2$$

roughly speaking $p_1 = 1$ deals with patches used as sensors, whereas $p_1 = 2$ concerns actuators (see [4,5]). The index p_2 accounts for the states of the interface between the piezoelectric inclusions and the body, $p_2 = 1$ corresponds to an electrically impermeable interface, $p_2 = 2$ corresponds to a grounded interface:

$$D^s n = 0 \quad \text{on } S_I^\varepsilon \quad (6)_1$$

$$\varphi^s = 0 \quad \text{on } S_I^\varepsilon. \quad (6)_2$$

Finally, we assume that s takes a value in a countable set with a sole cluster point such that $p_3 = 1, 2, 3$ if and only if $\lim_{s \rightarrow 0} (h/\varepsilon) = 0, 1, +\infty$, respectively.

It will be convenient to use the following notations

$$\begin{cases} \hat{k} = (\hat{e}, \hat{g}) & \hat{e} = (e_{\alpha\beta})_{\alpha, \beta \in \{1, 2\}} & \hat{g} = (g_1, g_2) & \forall k = (e, g) \in \mathbb{K} \\ k(r) = k(v, \psi) := (e(v), \nabla \psi) & \forall r = (v, \psi) \in H^1(B_I^s; \mathbb{R}^3) \times H^1(B_I^s) \end{cases} \quad (7)$$

An electromechanical state will be an element $r = (v, \psi)$ of

$$V_{\hat{p}} := H_{\Gamma_0}^1(\mathcal{O}^h; \mathbb{R}^3) \times \Psi_{\hat{p}}, \quad \Psi_{(1,1)} = H_{m_s}^1(B_I^s), \quad \Psi_{(1,2)} = H_{S_I^\varepsilon}^1(B_I^s), \quad \Psi_{(2,1)} = H_{S_I^s}^1(B_I^s), \quad \Psi_{(2,2)} = H_{S_I^\varepsilon \cup S_I^s}^1(B_I^s) \quad (8)$$

where for any domain $\mathcal{G}$ of $\mathbb{R}^3$, $H_\Gamma^1(\mathcal{G}; \mathbb{R}^3)$ and $H_\Gamma^1(\mathcal{G})$ respectively denote the subspaces of $H^1(\mathcal{G}; \mathbb{R}^3)$ and $H^1(\mathcal{G})$ of all elements with vanishing traces on a part Γ of $\partial \mathcal{G}$, while $H_{m_s}^1(B_I^s) := \left\{ \varphi \in H^1(B_I^s); \int_{\varepsilon(i+Y^*)} \varphi(x) d\hat{x} = 0 \quad \forall i \in I_\varepsilon \right\}$.

One makes the following assumptions on the data:

$$\begin{cases} \varphi_0^\varepsilon \text{ denotes the restriction to } S \text{ of an element of } H^{1/2}(\{x_3 = 0\}) \text{ still denoted by } \varphi_0^\varepsilon \\ (f, \mathcal{F}, q^\varepsilon) \in L^2(\Omega; \mathbb{R}^3) \times L^2(\Gamma_1; \mathbb{R}^3) \times L^2(S), \int_{\varepsilon(i+Y^*)} q^\varepsilon(\hat{x}) d\hat{x} = 0 \quad \forall i \in I_\varepsilon \text{ when } \hat{p} = (1, 1) \\ q^s(x + h e_3) = q^\varepsilon(x), \quad \varphi_0^s(x + h e_3) = h \varphi_0^\varepsilon(\hat{x}) \text{ a.e. } x \in S_I^\varepsilon \end{cases} \quad (H_1)$$

It is well known that for all φ_0^ε in $H^{1/2}(\{x_3 = 0\})$, there exists an element of $H^1(S \times (-L, 0))$ when $p_2 = 1$, of $H_{S-L}^1(S \times (-L, 0))$ when $p_2 = 2$, still denoted by φ_0^ε , whose trace on S is φ_0^ε . Hence the element φ_{0p}^s of $\Psi_{\hat{p}}$ defined by $\varphi_{0p}^s(x) = h \varphi_0^\varepsilon(\hat{x}, (x_3 - h)L/h)$ satisfies

$$\varphi_{0p}^s = \varphi_0^s \text{ on } S_I^s, \quad \frac{1}{h} \int_{B_I^s} |\nabla \varphi_{0p}^s|^2 dx \leq C \quad (9)$$

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