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Bridging meso- and microscopic anisotropic unilateral damage formulations for microcracked solids

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ABSTRACT

A mathematically consistent and unified description of induced anisotropy and unilateral effects constitutes one of the central tasks in the continuum damage theories developed so far. This paper aims at bridging constitutive damage formulations on meso- and micro-scales with an emphasis on a complete mesoscopic determination of material effective properties for microcracked solids. The key is to introduce a new set of invariants in terms of strain tensor and fabric tensor by making use of the Walpole's tensorial base. This invariant set proves to be equivalent to the classical one, while the new one provides great conveniences to high-order orientation-dependent tensor manipulations. When limited to the case of parallel microcracks, potential relations between ten combination coefficients are established by applying continuity conditions. It is found that the dilute approximation with penny-shaped microcracks is a particular case of the present one. By originally introducing effective strain effect, interactions between microcracks are taken into account with comparison to the Mori–Tanaka method as well as the Ponte-Castaneda and Willis scheme. For completeness, discussions are also addressed on macroscopic formulations with high-order damage variables.

1. Introduction

Microcracking is commonly viewed as the main dissipative mechanism governing the nonlinear mechanical behaviors of quasi-brittle materials like concrete and quasi-brittle rocks [1–3]. In constitutive modeling, the concept of continuum damage has been widely used within the framework of irreversible thermodynamics [4–6]. During the development of damage models, in view of the specific features of microcracks, two essential theoretical issues have attracted much attention and still remain largely open up to now: the first one is the induced anisotropy due to crack growth, nonuniform but in some preferred orientations; the second one, known as unilateral effect, is related to the unilateral contact between the lips of microcracks, which can be open or closed without material interpenetration. Concerning the mechanical phenomena related to the two mechanisms, we can mention the asymmetry of material strength in tension and in compression, the non-linearity of the material response, as well as the effect of confining pressure, volumetric dilatancy, occurrence of unloading–reloading hysteresis in the context of damage–friction coupling at closed cracks.

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Mathematically consistent description of cracks-related material anisotropy and unilateral effects is an essential and important theoretical issue in Continuum Damage Mechanics. At the early stage, four main methods were explored based on i) two scalar variables, ii) two second-order damage variables, iii) a fourth-order damage variable, and iv) vectorial damage variables, respectively [6,7]. Chaboche [7] discussed and compared the predictive capability of these theories by using a 2D damage problem and setting up four assessment rules. He found that there exist problems of some sort for all the four candidates. On the same topic, by relating macroscopic formulations to mesostructural mechanisms, Halm and Dragon [8] then developed a mesoscopic damage model by employing simultaneously a second-order variable and a fourth order variable; the latter one proved to be mandatory for describing the unilateral effects. However, in the Halm and Dragon's model, both the second- and fourth-rank damage variables have been formulated in the principal directions determined by spectral decomposition. Although such a choice avoided some theoretical difficulties, it brought about some inconsistencies in constitutive damage formulations and the non-objectivity of the mechanical response, as commented by Cormery and Welemane [9]. Welemane and Cormery [10] also made their own contribution to this issue with the help of the tensor representation theory [11] and the piecewise linearity theory [12]. In their work, some basic relations between the combination coefficients have been established by applying the continuity conditions for multilinear functions and the recovery condition of the elongation modulus in the normal direction of cracks. In spite of these significant advances, in-depth developments along this line are still required. On one hand, even in the case of dilute microcracks, the effective stiffness tensor reported by Welemane and Cormery [10] contains some terms whose physical meanings are not clear. On the other hand, there is still a need to establish some trans-scale relationships between the constitutive equations.

Concerning induced anisotropy and unilateral effects, we should mention the school of investigators whom account for unilateral effects by means of the spectral decomposition upon either the stress tensor or the strain tensor. To the authors' knowledge, the choice to separate the stress/strain tensors into positive and negative parts in the principal space can be traced back to the pioneering work by Ortiz [13], mainly based on the hypothesis that damage in quasi-brittle materials is induced by local tensile stress. In that context, the constitutive damage formulations were naturally closely related to principal stress/strain components. It is noted that the Oritz's theory has been followed by numerous authors, particularly those in concrete researches. We mention, among others, the works with stress decomposition [5,14–20] and those using strain decomposition [21–24]. With the spectral decomposition technique, when limited to tensile damage, modeling of nonlinear behaviors of quasi-brittle materials like concrete has gained great success in view of their practical applications. However, this method exhibits obvious disadvantages on both theoretical and numerical aspects: firstly, because of the neglect of the intrinsic fabric by microcracks, the theoretical foundation of the use of stress/strain splitting is not firm and the piecewise linearity for general loading paths has not been proved. To a large extent, the models framed by the spectral decomposition technique should be classed into the category of phenomenological models; secondly, for nonlinear problems, the frequent numerical resolution of the eigenvalue problem implies high computational costs, as commented by Curnier et al. [12], and it is thus not in favor of parallel programming and computing.

The present work is motivated partially by the current situation of the development of the continuum damage mechanics. During more than two decades, the phenomenological damage models and the micromechanical damage models have been investigated individually. The former describes cracking-induced damage by using tensorial internal variables [6]. It is noticed in the literature that the isotropic damage models are still playing a predominant role in practical applications although they exhibit obvious disadvantages in accounting for the mechanical mechanisms behind the main phenomena observed at laboratory. Starting from the microstructure and local behaviors of cracked media, the micromechanical damage models aims at formulating constitutive equations by means of some upscaling methods, such as the homogenization technique. In recent years, these approaches have experienced a great success and have paved a promising way to deliver a mechanisms-based modeling of quasi-brittle materials. However, when compared to the phenomenological models, the mathematical tools used and the constitutive formulations presented in the micromechanical models are somewhat complex, especially for researchers who are mainly concerned with engineering applications. Up to now, few efforts have been devoted to building some links between these two categories of damage models. This work attempts to get out of this trouble and to fill in this gap by dealing with the two critical theoretical issues in continuum damage mechanics.

Along the line of the work by Welemane and Cormery [10], this paper delivers new contributions to the research issues discussed above. Attempts were first made to determine the explicit form of the effective properties for a matrix-cracks system. We start with the classical set of strain-damage invariants largely used in the macroscopic and mesoscopic damage models. For convenience, a new set of invariants is constructed based on the Walpole tensorial base and its equivalence to the classical one is proved. Induced anisotropies and unilateral damage are taken into account in a unified and consistent way. Potential relations between the coefficients companying with the invariants for both the cases of open cracks and closed cracks are explored step by step in Section 3. Efforts in Sections 4 and 5 are addressed to two aspects: i) compare the mesoscopic formulations with the micromechanical results obtained by applying some popular homogenization schemes [25–29], ii) address the theoretical issues with high-order damage variables. The difficulties are revealed and a possible solution is discussed. Finally, in Section 6 are given some concluding remarks.

2. Introduction of an alternative set of invariants

We are first concerned with the case of a solid matrix weakened by one unique family of parallel non-interacting microcracks. The extension to the general case of multiple crack families will be treated in Section 4. Cracked solids are viewed as