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Evolutionary fracture analysis of masonry arches: Effects of shallowness ratio and size scale

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A B S T R A C T

Masonry arch structures can be determined by means of a detailed analysis that takes into account the intermediate cracking stage, which takes place when the tensile strength of the material has been exceeded, even though the collapse mechanism has not formed yet. Such a hypothesis is based on a constitutive law that returns a closer approximation to the actual material's behaviour.

This paper presents the evolutionary analysis for the fracturing assessment of masonry arches. This method allows capturing the damaging process that occurs when the linear elastic behaviour's conditions in tension no longer apply, and before achieving the limit conditions. Furthermore, the way the thrust line is influenced by the formation of cracks and the consequent internal stresses redistribution, representing the "fracturing benefit", can be assessed numerically. Size scale effects are also taken into account, as well as the influence of the arch's shallowness ratio.

1. Fracturing process in masonry arches

Masonry is characterised by both anisotropic and nonlinear behaviour; such behaviours are detected even at low strain values [1–3]. When undergoing uniaxial loading tests, masonry shows appreciably different values relating to tensile and compressive strength: the latter results to be significantly higher than the former.

The elastic-softening constitutive law is that which best represents the natural or artificial masonry behaviour. This corresponds to simply considering an elastic constitutive law associated with a fracturing crisis condition consistent with the concepts of Linear Elastic Fracture Mechanics (LEFM). This means that the material has merely elastic behaviour with the possibility that cracks might form and propagate [4,5].

The crack depth $\xi = a/b$ (Fig. 1a), as well as the stress intensity factor, K_{I} (Fig. 1b) will be taken into consideration as damage parameter and load parameter respectively. The Mode-I stress intensity factor can be considered a stress field's amplification factor when the loads are symmetrical to the crack (e.g., axial force and bending moment) [4–7].

Shear is disregarded [8–11]. The validity of this assumption, taking into account the Mery's theory [8], and the Heyman's hypotheses of Limit Analysis [10], can be verified considering the slope of the arch thrust line with respect to the joint lines. If the thrust line affects the joints with a slope less than the angle of friction, no mutual sliding takes place between two adjacent elements [11]. Also by a LEFM point of view, the presence of the compressive stresses in the arch structure reduces

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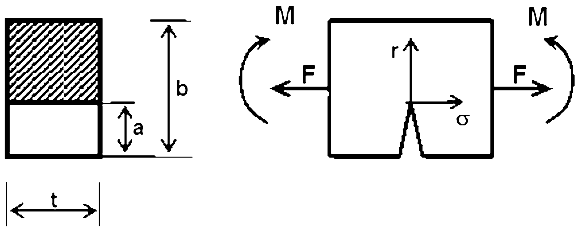

Fig. 1. Cracked beam element: $\xi = a/b$; $\sigma = K_\mathrm{I}(2\pi r)^{-0.5}$.

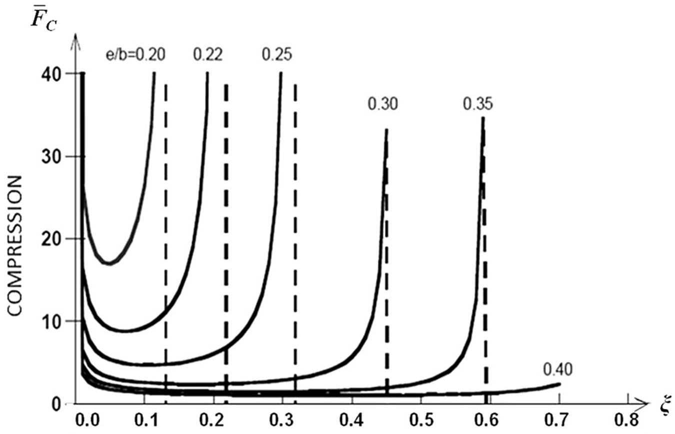

Fig. 2. Fracturing process for eccentric axial load.

the risk of Mode-II failure (shear). Due to the contact stresses between the crack surfaces, friction is caused, resulting in a Mode-II stress intensity factor contribution $K_{\mathrm{II,frict}}$ [12]. Then, the effective Mode II stress intensity factor is:

$$K_{\mathrm{II,eff}} = K_{\mathrm{II,appl}} - K_{\mathrm{II,frict}} < K_{\mathrm{IIC}}$$

where $K_{\mathrm{II,appl}}$ is the Mode-II stress intensity factor depending on shear loading [12].

With a compressive axial force, and when the bending moment opens the crack, as is usually the case with masonry arches, it is possible to determine the total stress intensity factor by means of the Superposition Principle [4,5]:

$$K_\mathrm{I} = K_{\mathrm{I}M} - K_{\mathrm{I}F} = \frac{M}{tb^{3/2}} Y_M(\xi) - \frac{F}{tb^{1/2}} Y_F(\xi) = \frac{F}{tb^{1/2}} \left[\frac{e}{b} Y_M(\xi) - Y_F(\xi) \right] \tag{1}$$

where $K_{\mathrm{I}M}$ is the stress intensity factor for pure bending $M = Fe$, $K_{\mathrm{I}F}$ is the stress intensity factor for the compressive axial force F, and e is the equivalent eccentricity of the axial force, relative to the cross-sectional area's centroid. The sign minus in Eq. (1) based on the Superposition Principle is due to the effect of the compressive axial force, which tends to close the crack, while the bending moment opens the crack. Moreover, as can be found in [7], $Y_M(\xi)$, $Y_F(\xi)$ represent the shape functions for $K_{\mathrm{I}M}$ and $K_{\mathrm{I}F}$, respectively.

The critical condition $K_\mathrm{I} = K_{\mathrm{IC}}$ allows determining two factors: on the one hand, the dimensionless crack extension axial force as crack depth's function ξ; on the other one, the load's relative eccentricity, e/b:

$$\bar{F}_\mathrm{C} = \frac{F_\mathrm{C}}{tb^{1/2} K_{\mathrm{IC}}} = \frac{1}{\frac{e}{b} Y_M(\xi) - Y_F(\xi)} \tag{2}$$

Equation (2) is graphically represented by the curves in Fig. 2, which also show how, with a fixed eccentricity e/b, the fracturing process becomes stable only after showing a condition of instability. If the load F does not follow the decreasing unstable branch in strain-softening unloading processes along an $e/b =$ constant curve, then the fracturing process will show a catastrophic behaviour: the representative point will advance horizontally until meeting again the $e/b =$ constant curve situated on the stable branch (snap-through). Moreover, the possibility of load relaxation, as well as of a less catastrophic fracturing behaviour, is linked to the structure's geometry and mechanical characteristics. It is especially affected by both the degree of redundancy and the structural size [4,5].

It is also important to take into account that, for each relative crack depth ξ, there exists a relative eccentricity value; below such a value, the crack tends to close again, at least partially [4,5]. The closing condition $K_\mathrm{I} = 0$, leads to:

$$K_\mathrm{I} = 0 = \frac{F}{tb^{1/2}} \left[\frac{e}{b} Y_M(\xi) - Y_F(\xi) \right] \tag{3}$$