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Bifurcation analysis of a flexible balanced cracked rotor–bearing system

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The dynamic analysis of cracked rotors is of considerable current interest. In the present paper, the effect of the presence of a transverse crack in a rotor supported by two hydrodynamic journal bearings is investigated. A nonlinear model of a flexible cracked rotor–bearing system is proposed. The model of the hydrodynamic forces is derived based on the assumption of a short bearing approximation and a half-Sommerfeld boundary condition. The system of nonlinear differential equations is integrated numerically using the Runge–Kutta method. The effect of the crack depth on the motion of the journal center in the vicinity of the stability limit is investigated. Bifurcation diagrams and Poincaré maps are used to determine the effect of the crack on the stability limit and on the journal motion.

1. Introduction

A crack defect in a rotor is generally considered a sever fault. The dynamic analysis of cracked rotors is a problem of great interest due to its practical importance; consequently, it has received considerable attention in the last decades. The presence of a crack affects the dynamic response of a rotor because of the reduction in the rotor's stiffness and thus in the natural frequencies of the original uncracked rotor [1]. Different approaches have been proposed for dynamic analysis of a cracked rotor based on two established models. The first one is the model of opening and closing crack, or switching model. In this model, the stiffness of the rotor varies between the stiffness corresponding to the closed crack and the stiffness corresponding to the open crack. In the second model, known as the breathing crack model, a partial opening/closing of the crack is modeled using a periodic function that governs the change in stiffness. Several studies have considered the effect of a crack on the stability of the rotor using several methods to model the presence of a crack. Papadopoulos and Dimarogonas [2] used an analytical approach to compute the local flexibility of a cracked shaft using the strain energy and the stress intensity. This approach has been validated by experimental results. Sekhar and Prabhu [3] computed the natural frequencies and mode shapes of a cracked rotor using a Finite Element Model. The detection of a crack in the shaft is based on the variation of natural frequencies. Mayes and Davies [4] studied the presence of a crack using the switching crack model. The opening and closing of the crack are modeled by a truncated Fourier series. Mancilla and Sinou [5] showed, using a numerical simulation, that the evolution of the orbit of the journal center on the one-half and one-third first

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Nomenclature

m	rotor mass per bearing
g	gravity constant
$\{Z\} = \{x\ y\}^{\mathrm{T}}$	vector of the disk displacements in inertial coordinates
a	depth of the crack
d	diameter of the shaft
$r = a/d$	dimensionless crack depth ratio
l	length of the shaft
C	bearing radial clearance
$e = \overline{O_b O_j}$	bearing eccentricity
$\varepsilon = e/C$	relative eccentricity
ω	angular velocity
$\varpi = \omega\sqrt{C/g}$	dimensionless journal angular velocity
$\tau = \omega t$	dimensionless time
ϕ	attitude angle
β	torsional angle
ξ, η	rotating coordinates, ξ along crack orientation, η perpendicular to the crack orientation
O_{b}	bearing center
O_{j}	dynamic position of the journal center
f_ε, f_ϕ	radial and tangential components of the hydrodynamic forces
K	half shaft stiffness
$\Delta K_\xi, \Delta K_\eta$	stiffness changes due to crack, in ξ and η directions
Δk_ξ	dimensionless stiffness change along ξ direction ($\Delta k_\xi = \Delta K_\xi / K$)
Δk_η	dimensionless stiffness change along η direction ($\Delta k_\eta = \Delta K_\eta / K$)
$\bar{K} = K(C/mg)$	dimensionless stiffness of the uncracked shaft
$\Gamma = \mu R L^3/(2mC^{2.5} g^{0.5})$	bearing parameter
μ	constant lubricant viscosity
$g(\omega t)$	crack function that defines the angular position of the crack
k_{ij} $(i = x, j = y)$	stiffness matrix coefficients
$[K_{\mathrm{R}}]$	stiffness matrix of the rotor system in rotating coordinate system
$[K_{\mathrm{I}}]$	stiffness matrix of the rotor system in inertial coordinate system
$[M]$	rotor mass matrix

frequency can be used to detect rotor cracks. Sinou and Lees [6] evaluated the dynamic response of a rotor with a breathing crack by using the time-frequency domain approach. The increase in the harmonic components of the dynamic system response and the evolution of the size of the orbit are the principal indicators of the presence of a crack in a rotating shaft. Later, they have used the Harmonic Balance Method to study the influence of a crack on the dynamic response of a rotor. The evolution of the trajectory center of the rotor near half of the first critical speed is an indication of the existence of a crack [7]. Xue and Cao [8] used the Runge–Kutta method to solve a system of differential equations of a cracked rotor. Bifurcation diagrams, Poincaré sections and rotor trajectory diagrams are used to study the effect of rotating speed, and crack depth on the dynamic response of the rotor. The onset of a period doubling orbit is considered an indicator of the presence of a crack in the shaft. The domain of instability increases considerably when the crack deepens.

A nonlinear model of a cracked rotor–bearing system is used by Luo and Zhang [9]. Harmonic frequency method is used to study the presence of the crack. The analysis proves that the increase of the depth of a crack reduces the stability limit of the system. Chaofeng and Hexing [10] used the Finite Element Method to study the stability of a nonlinear cracked rotor–bearing system. To study the effect of the depth of the crack on the stability of the rotor as a function of the rotor speed, they used bifurcation diagrams, Poincaré sections, and trajectories of the shaft center. They also showed that the crack fault reduces the stability speed of the rotor as the crack becomes deeper. The simulation results have been validated experimentally. In this study, they showed that the stability speed limit decreases as the crack becomes deeper. Gasch [11] used Floquet theory to study the effect of a crack in the vibrations signal. He showed that a deeper crack in the shaft changes the natural frequencies, and that the crack detection, in practice, is difficult for cracks depth less than 25%.

In this paper, a four-degree-of-freedom dynamic model of a perfectly balanced flexible cracked rotor supported by hydrodynamic journal bearings is derived. The fluid film force components corresponding to the half Sommerfeld conditions for a short hydrodynamic bearing are used ($L/D \leq 0.25$). Bifurcation diagrams and trajectories of the journal center have been used to predict the effect of the crack depth on the limits of the stability and its motion in the vicinity of the stability limit. The validity of this research is proved by the detection of a crack in the journal using bifurcation diagrams, Poincaré sections, trajectories of the journal center and power spectra.

2. A nonlinear model of a balanced flexible cracked rotor–bearing system

Fig. 1 shows a symmetrical balanced flexible cracked rotor that has a mass $2m$ supported by two identical hydrodynamic short bearings O_{b} is the bearing center, O_{j} is the dynamic position of the journal center and f_ε, f_ϕ are respectively the radial and tangential components of the hydrodynamic forces applied on each journal bearing.

To analyze the dynamic response of a flexible balanced cracked rotor, the following assumptions are used:

(a) the deflection of the shaft is sufficiently small to permit the use of classical linear beam theory;
(b) due to the short bearing assumption and lubricant force description, the length to diameter, L/D, has to be less or equal to 0.25;