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# Homogenization of random attractors for reaction–diffusion systems ☆

## *Homogénéisation des attracteurs aléatoires pour les systèmes d'équations de réaction–diffusion*

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ABSTRACT

We consider reaction–diffusion systems with randomly oscillating terms. We construct the deterministic homogenized reaction–diffusion system and prove that the trajectory attractors of the initial systems converge to the trajectory attractors of the homogenized systems.


RÉSUMÉ

Nous considérons les systèmes d'équations de réaction–diffusion avec termes aléatoirement oscillants. Nous construisons le système homogénéisé déterministe d'équations et prouvons que les attracteurs trajectoires des systèmes initiaux convergent vers les attracteurs trajectoires des systèmes d'équations homogénéisées.


## 1. Introduction

In this paper, we study an asymptotic behavior of attractors of the reaction–diffusion systems with randomly oscillating terms. To study such a phenomenon, we apply the homogenization method (cf., for example, [1–7], for the random case

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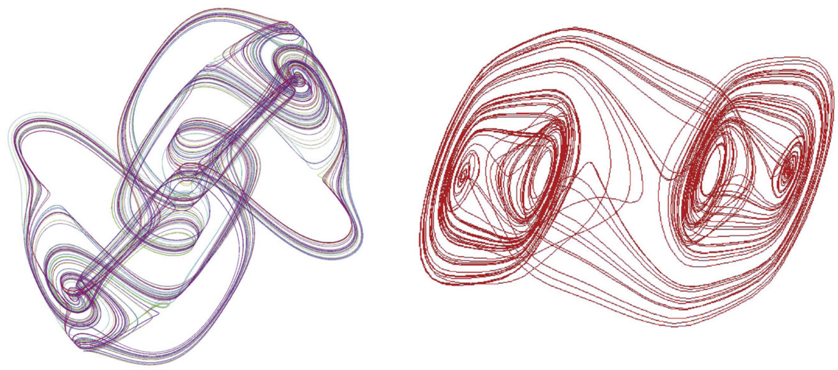

**Fig. 1.** Thomas' cyclically symmetric attractor (Model: Clint Sprott) and a 4-spiral strange attractor exhibited by the modified Chua's circuit (Model: M.A. Aziz Alaoui).

cf., for instance, [8–11]), as well as a delicate analysis of trajectory and global attractors (see, for example, [12–14] and references therein), see Fig. 1.

In this paper, we prove that the trajectory attractor $\mathfrak{A}_\varepsilon$ of the autonomous reaction–diffusion system with a randomly oscillating term converges almost surely as $\varepsilon \to 0$ to the trajectory attractor $\overline{\mathfrak{A}}$ of the homogenized reaction–diffusion system in an appropriate functional space.

## 2. Homogenization

Assume that $(\Omega, \mathcal{B}, \mu)$ is a probability space, i.e. the set $\Omega$ is endowed with a $\sigma$-algebra $\mathcal{B}$ of its subsets and a $\sigma$-additive nonnegative measure $\mu$ on $\mathcal{B}$ such that $\mu(\Omega) = 1$.

We consider the system of reaction–diffusion equations with randomly oscillating terms of the form

$$\partial_t u = a\Delta u - b\left(x, \frac{x}{\varepsilon}, \omega\right) f(u) + g\left(x, \frac{x}{\varepsilon}, \omega\right), \ u|_{\partial D} = 0 \tag{1}$$

where $x \in D \Subset \mathbb{R}^n$, $u = (u^1, \dots, u^N)$, $f = (f^1, \dots, f^N)$, and $g = (g^1, \dots, g^N)$. Here $a$ is an $N \times N$ matrix with positive symmetric part and $b(x, z, \omega) \in C(D \times \mathbb{R}^N \times \Omega)$ is a real positive function. The Laplace operator $\Delta := \partial^2_{x_1} + \ldots + \partial^2_{x_n}$ acts in $x$-space.

We note that all the results can be extended to the systems with nonlinear terms of the form $\sum_{j=1}^{m} b_j\left(x, \frac{x}{\varepsilon}, \omega\right) f_j(u)$, where $b_j$ are positively defined matrices and $f_j(u)$ are vector functions. For brevity, we consider the case $m = 1$ and $b_1\left(x, \frac{x}{\varepsilon}, \omega\right) = b\left(x, \frac{x}{\varepsilon}, \omega\right) I$, where $I$ is the identity matrix and $b$ is a real function.

For the sake of simplicity, we assume that the vector function $f(v) \in C(\mathbb{R}^N; \mathbb{R}^N)$ satisfies the following inequalities:

$$f(v) \cdot v \geq \gamma |v|^p - C, \ |f(v)| \leq C_1\left(|v|^{p-1} + 1\right), \ p \geq 2 \tag{2}$$

Notice that we *do not assume* that the function $f(v)$ satisfies the Lipschitz condition with respect to $v$.

Assume that $T_\xi$, $\xi \in \mathbb{R}^n$, is an ergodic dynamical system. The function $b(x, \frac{x}{\varepsilon}, \omega)$ and the vector function $g(x, \frac{x}{\varepsilon}, \omega)$ are statistically homogeneous, i.e. $b(x, \xi, \omega) = \mathbf{B}(x, T_\xi \omega)$ and $g(x, \xi, \omega) = \mathbf{G}(x, T_\xi \omega)$, where $\mathbf{B} : D \times \Omega \to \mathbb{R}$ and $\mathbf{G} : D \times \Omega \to \mathbb{R}^N$ are measurable.

We also assume that $b(x, z, \omega) \in C_b(\overline{D} \times \mathbb{R} \times \Omega)$ and

$$\beta_1 \geq b(x, z, \omega) \geq \beta_0 > 0, \ \forall x \in D, \ z \in \mathbb{R}^n, \ \omega \in \Omega \tag{3}$$

the function $b\left(x, \frac{x}{\varepsilon}, \omega\right)$ has the average $b^{\mathrm{hom}}(x) = \mathbb{E}(\mathbf{B})(x)$ as $\varepsilon \to 0+$ in $L_{\infty,*w}(D)$, that is, almost surely

$$\int\limits_D b\left(x, \frac{x}{\varepsilon}, \omega\right) \varphi(x)\,\mathrm{d}x \to \int\limits_D b^{\mathrm{hom}}(x)\, \varphi(x)\,\mathrm{d}x \ (\varepsilon \to 0+) \tag{4}$$

for any function $\varphi \in L_1(D)$. For the vector function $g\left(x, \frac{x}{\varepsilon}, \omega\right)$, we assume that it has the average $g^{\mathrm{hom}}(x) = \mathbb{E}(\mathbf{G})(x)$ in the space $V' = \left(H^{-1}(D)\right)^N$:

$$g\left(x, \frac{x}{\varepsilon}, \omega\right) \rightharpoonup g^{\mathrm{hom}}(x) \ (\varepsilon \to 0+) \text{ weakly in } V'$$

that is, almost surely

$$\left\langle g\left(x, \frac{x}{\varepsilon}, \omega\right), \varphi(x)\right\rangle \to \left\langle g^{\mathrm{hom}}(x), \varphi(x)\right\rangle \ (\varepsilon \to 0+) \tag{5}$$

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