C. R. Mecanique 344 (2016) 736–743

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Transient response of elastic bodies connected by a thin stiff viscoelastic layer with evanescent mass

Réponse transitoire de corps élastiques liés par une mince et raide bande viscoélastique de faible masse

Christian Licht [a,b,c], Somsak Orankitjaroen [b,c], Ahmed Ould Khaoua [d], Thibaut Weller [a,*]

[a] *LMGC, UMR CNRS 5508, Université Montpellier-2, case courier 048, place Eugène-Bataillon, 34095 Montpellier cedex 5, France*
[b] *Department of Mathematics, Faculty of Science, Mahidol University, Bangkok 10400, Thailand*
[c] *Centre of Excellence in Mathematics, CHE, Bangkok 10400, Thailand*
[d] *Departamento de Matemáticas, Universidad de los Andes, Cra 1 No 18A-12, Bogota, Colombia*

A R T I C L E   I N F O

Article history:
Received 16 June 2016
Accepted 6 July 2016
Available online 9 August 2016

Keywords:
Bonding problems
Kelvin–Voigt viscoelasticity
Dynamics
Maximal monotone operators

Mots-clés :
Problèmes de collage
Viscoélasticité de Kelvin–Voigt
Dynamique
Opérateurs maximaux monotones

A B S T R A C T

We extend the study [1] devoted to the dynamic response of a structure made up of two linearly elastic bodies connected by a thin soft adhesive layer made of a Kelvin–Voigt-type nonlinear viscoelastic material to the cases of stiff and very stiff adhesives whose mass vanishes. We use a nonlinear extension of Trotter's theory of convergence of semi-groups of operators acting on variable spaces to identify the asymptotic behavior of the mechanical state of the system, when some geometrical and mechanical parameters tend to their natural limits. The models we obtain describe the behavior of a structure consisting of two linearly elastic adherents perfectly bonded to a material deformable flat surface whose behavior is of the same kind as that of the genuine adhesive.

R É S U M É

Nous étendons aux adhésifs durs ou très durs, dont la masse est évanescente, l'étude menée en [1] consacrée au comportement dynamique d'un assemblage de deux corps linéairement élastiques liés par une couche adhésive mince et molle constituée d'un matériau viscoélastique non linéaire de type Kelvin–Voigt. Afin d'identifier le comportement asymptotique de l'état mécanique du système lorsque des paramètres mécanique et géométriques tendent vers leurs limites naturelles, nous utilisons une extension non linéaire de la théorie de Trotter de convergence de semi-groupes d'opérateurs agissant sur des espaces variables. Les modèles obtenus décrivent le comportement d'une structure constituée de

* Corresponding author.
E-mail addresses: clicht@univ-montp2.fr (C. Licht), somsak.ora@mahidol.ac.th (S. Orankitjaroen), aould@uniandes.edu.co (A. Ould Khaoua), thibaut.weller@umontpellier.fr (T. Weller).

http://dx.doi.org/10.1016/j.crme.2016.07.001

deux adhérents élastiques parfaitement collés à une surface matérielle plate et déformable, dont le comportement est identique à celui de l'adhésif.

1. Setting the problem

We extend to the situations of high and very high stiffness the results obtained in [1] concerning the dynamics of elastic bodies connected by a thin soft viscoelastic layer. Let $\{e_1, e_2, e_3\}$ be an orthonormal basis of $\mathbb{R}^3$ assimilated to the Euclidean space. For all $\xi = (\xi_1, \xi_2, \xi_3)$ in $\mathbb{R}^3$, $\widehat{\xi}$ stands for (ξ_1, ξ_2). The space of all $(n \times n)$ symmetric matrices is denoted by $\mathbb{S}^n$ and equipped with the usual inner product and norm denoted by $\cdot$ and $|\ |$ (as in $\mathbb{R}^3$). For all η in $\mathbb{S}^3$, $\widehat{\eta}$ stands for the matrix $(\eta_{\alpha\beta})_{1\le\alpha,\beta\le 2}$ in $\mathbb{S}^2$. We study the dynamic response of a structure consisting of two adherents connected by a thin adhesive layer which is subjected to a given loading. Let Ω be a domain of $\mathbb{R}^3$ with Lipschitz-continuous boundary $\partial\Omega$. The intersection of Ω with $\{x_3 = 0\}$ is a domain S of $\mathbb{R}^2$ with a positive two-dimensional Haussdorf measure $\mathcal{H}_2(S)$. Let ε be a positive number and $\Omega_\pm := \Omega \cap \{\pm x_3 > 0\}$, then adhesive and adherents occupy $B^\varepsilon := S \times (-\varepsilon, +\varepsilon)$ and $\Omega^\varepsilon_\pm := \Omega_\pm \pm \varepsilon e_3$ respectively; we define $\Omega^\varepsilon := \Omega^\varepsilon_+ \cup \Omega^\varepsilon_-$, $S^\varepsilon_\pm := S \pm \varepsilon e_3$ and $\mathcal{O}^\varepsilon := \Omega^\varepsilon \cup B^\varepsilon \cup S^\varepsilon_+ \cup S^\varepsilon_-$. We consider a partition (Γ_0, Γ_1) of $\partial\Omega$ and, for all Γ in $\{\Gamma_0, \Gamma_1\}$, the sets $\Gamma_\pm$, $\Gamma^\varepsilon_\pm$ and Γ^ε respectively denote $\Gamma \cap \{\pm x_3 > 0\}$, $\Gamma_\pm \pm \varepsilon e_3$ and $\Gamma^\varepsilon_+ \cup \Gamma^\varepsilon_-$. Moreover, we assume that $\mathcal{H}_2(\Gamma_{0+}) > 0$. The structure made of the adhesive and the two adherents, perfectly stuck together along $S^\varepsilon_\pm$, is clamped on Γ^ε_0 and subjected to body forces of density f^ε and to surface forces g^ε on Γ^ε_1. The adherents are modeled as linearly elastic materials with a strain energy density W^ε such that

$$W^\varepsilon(x, e) = \frac{1}{2} a^\varepsilon(x) e \cdot e, \quad a.e.\, x \in \Omega^\varepsilon, \quad \forall e \in \mathbb{S}^3 \tag{1}$$

The *thin* adhesive is assumed to be made of a homogeneous, isotropic and "viscoelastic of Kelvin–Voigt generalized type". Its strain energy density reads as $\mu\, w_I$, while its dissipation potential is denoted by $b\,\mathcal{D}$, where μ and b are positive scalars; w_I is a positive definite quadratic form on $\mathbb{S}^3$ and $\mathcal{D}$ a convex and positively homogeneous function of degree q, $1 \le q \le 2$.

Let $\rho > 0$, $\overline{\rho}_M > \overline{\rho}_m > 0$ and $\overline{\rho}^\varepsilon$ a measurable function. The density γ^ε of the structure is equal to $\overline{\rho}^\varepsilon$ in Ω^ε and to ρ in B^ε. Denoting by $\mathrm{Lin}(\mathbb{S}^3)$ the space of linear mappings from $\mathbb{S}^3$ into $\mathbb{S}^3$, we make the following assumptions on the data:

$$\begin{cases} \text{There exists } (f, g, a, \overline{\rho}) \text{ in } L^2(\Omega; \mathbb{R}^3) \times L^2(\Gamma_1; \mathbb{R}^3) \times L^\infty(\Omega; \mathrm{Lin}(\mathbb{S}^3)) \times L^\infty(\Omega) \text{ such that} \\ f^\varepsilon(x) = f(x \mp \varepsilon e_3) \quad a.e.\, x \in \Omega^\varepsilon_\pm, \quad f^\varepsilon(x) = 0 \quad a.e.\, x \in B^\varepsilon \\ g^\varepsilon(x) = g(x \mp \varepsilon e_3) \quad a.e.\, x \in (\Gamma_1)^\varepsilon_\pm, \quad g^\varepsilon(x) = 0 \quad a.e.\, x \in \partial S \times (-\varepsilon, \varepsilon) \\ a^\varepsilon(x) = a(x \mp \varepsilon e_3) \quad a.e.\, x \in \Omega^\varepsilon_\pm \\ \overline{\rho}^\varepsilon(x) = \overline{\rho}(x \mp \varepsilon e_3) \quad a.e.\, x \in \Omega^\varepsilon_\pm \\ \exists a_m, a_M > 0 \quad s.t. \quad a_m |e|^2 \le a(x) e \cdot e \le a_M |e|^2, \quad \forall e \in \mathbb{S}^3 \\ \exists \overline{\rho}_m, \overline{\rho}_M > 0 \quad s.t. \quad \overline{\rho}_m \le \overline{\rho}(x) \le \overline{\rho}_M, \quad a.e.\, x \in \Omega \end{cases} \tag{2}$$

Thus, the problem $(\mathcal{P}_s)$ of determining the dynamic evolution of the assembly involves a quadruplet $s := (\varepsilon, \mu, b, \rho)$ of data so that all the fields will be hereafter indexed by s. In the following, t denotes the time, $e(u)$ is the linearized strain tensor associated with the field of displacement u, and $\partial J(v)$ denotes the subdifferential at v of any lower semi-continuous convex function J, while $DJ(v)$ stands for the differential at v of any Fréchet differentiable function J. If $U^0_s = (u^0_s, v^0_s)$ is the initial state, a formulation of $(\mathcal{P}_s)$ could be

$$(\mathcal{P}_s) \quad \begin{cases} \text{Find } u_s \text{ sufficiently smooth in } \Omega \times [0, T] \text{ such that } u_s = 0 \text{ on } \Gamma^\varepsilon_0 \times (0, T] \\ \Big(u_s(\cdot, 0), \dfrac{\partial u_s}{\partial t}(\cdot, 0)\Big) = U^s_0 \text{ and there exists } \zeta \text{ in } \partial\mathcal{D}(e(\dfrac{\partial u_s}{\partial t})) \text{ satisfying:} \\ \displaystyle\int_{\mathcal{O}^\varepsilon} \gamma^\varepsilon \frac{\partial^2 u_s}{\partial t^2} v \,\mathrm{d}x + \int_{\Omega^\varepsilon} a^\varepsilon e(u_s) \cdot e(v) \,\mathrm{d}x + \int_{B^\varepsilon} \Big(\mu\, D w_I(e(u_s)) + b\zeta\Big) \cdot e(v) \,\mathrm{d}x = \\ \displaystyle\qquad = \int_{\mathcal{O}^\varepsilon} f^\varepsilon \cdot v \,\mathrm{d}x + \int_{\Gamma^\varepsilon_1} g^\varepsilon \cdot v \,\mathrm{d}\mathcal{H}_2 \\ \text{for all } v \text{ sufficiently smooth in } \mathcal{O}^\varepsilon \text{ and vanishing on } \Gamma^\varepsilon_0 \end{cases}$$

2. Existence and uniqueness

We assume that

$$(f, g) \in BV\big(0, T; L^2(\Omega; \mathbb{R}^3)\big) \times BV^{(2)}\big(0, T; L^2(\Gamma_1; \mathbb{R}^3)\big) \tag{$\mathbf{H}_1$}$$