International Journal of Engineering Science 121 (2017) 1–13

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

On contact problem with finite friction for a graded piezoelectric coating under an insulating spherical indenter

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ARTICLE INFO

Article history:
Received 21 November 2016
Revised 11 June 2017
Accepted 9 August 2017

Keywords:
Functionally graded piezoelectric material
Finite friction
Coating
Coupled singular integral equations

ABSTRACT

In this paper, the partial slip behavior of a spherical indenter on a functionally graded piezoelectric coating attached to a piezoelectric substrate is considered. The electromechanical constants of the functionally graded piezoelectric coating vary as the exponential gradation form within the thickness-coordinate. The partial slip contact problem is reduced to a system of singular integral equations with Cauchy kernels by making use of the Hankel transform technology. To solve the coupled singular integral equations, an approximate solution method is employed. The impact of the gradient index, friction coefficient and different piezoelectric substrates on the normal and radial tangential tractions are analyzed and discussed.

1. Introduction

In order to increase the lifetime and the reliability of the micro-electro-mechanical systems (MEMS) and smart structures, functionally graded piezoelectric materials (FGPMs) (Wu, Kahn, & Moy, 1996), whose mechanical properties are similar to FGMs, are introduced. Since the mechanical and electrical properties of FGPMs vary continuously, they can not only generate large output displacements but also reduce the stress concentrations and increase the bonding strength (Zhu & Meng, 1995).

Since indentation test is an effective method for the characterization of the mechanical and electrical properties of piezoelectric thin films, many researchers have focused their attention on the indentation problems of piezoelectric materials in the recent years. Giannakopoulos and Suresh (1999) presented the general theory for the indentation problems of piezoelectric materials. They analyzed the mechanical and electrical responses of a piezoelectric half-space under axisymmetric indenters, where the different electrical boundary conditions for electrically conducting or insulating indenters have been considered. Experimental studies of indentation problems for piezoelectric solids were conducted by Sridhar, Giannakopoulos, and Suresh (2000). They proved that the instrumented cone indentation is a valuable method in assessing the mechanical and electrical responses of piezoelectric materials in small volumes. Chen, Shioya, and Ding (1999) and Chen (2000) used the potential theory to study the contact problem for transversely isotropic piezoelectric materials under the conical punch. By using the technique of Hankel transformation, Wang, Chen, and Lu (2008) and Wang and Chen (2011) obtained the closed-form solutions for the frictionless indentation responses of a piezoelectric film/rigid substrate system under axisymmetric indenters. Luis, Andrés, and Aliabadi (2016) analyzes piezoelectric finitely thick and thin films under frictional indentation condition with the boundary element method. In the aforementioned previous studies, the contact problem

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http://dx.doi.org/10.1016/j.ijengsci.2017.08.001
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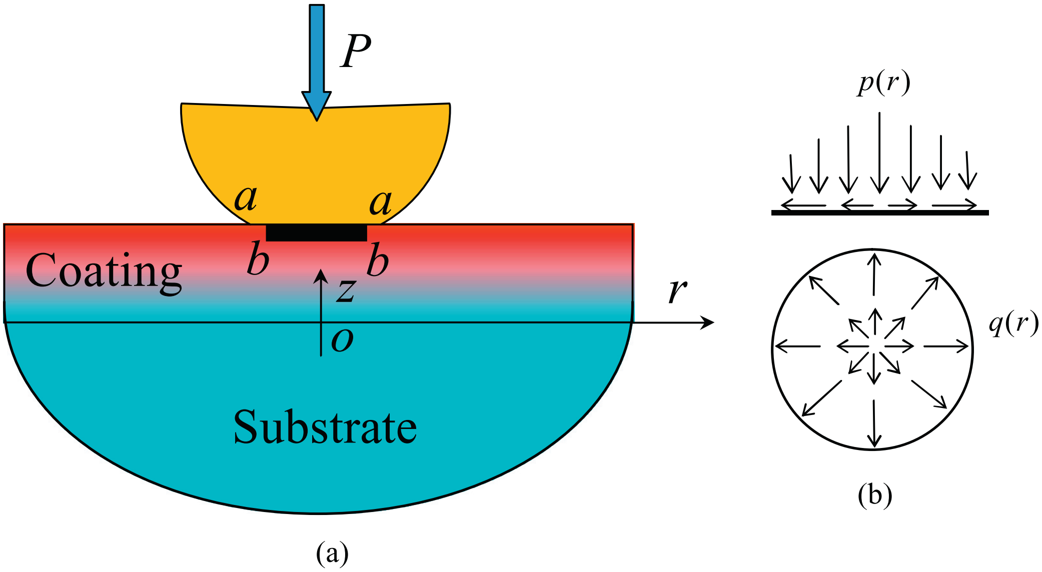

Fig. 1. The functionally graded piezoelectric coating/substrate system indented by a rigid insulating spherical indenter (a) and axisymmetric distribution of the normal and radial tractions (b).

of the FGPMs has not been tackled. Recently, Ke, Yang, Kitipornchai, and Wang (2008) and Ke, Wang, Kitipornchai, and Yang (2010) solved the two-dimensional (2D) contact problems of an FGPM layered half-plane with the Fourier integral transform. Liu, Zhang, and Wang (2016) and Liu and Zhang (2016) investigated the three-dimensional (3D) axisymmetric contact problems of the FGPM coating-substrate system under the insulating and conducting indenter by using the Hankel integral transform. However, the previous investigations on the axisymmetric contact problems of the FGPM coating-substrate system did not consider the effects of frictions.

In this paper, the axisymmetric contact problem with a finite friction for a functionally graded piezoelectric coating under an insulating spherical indenter is considered. The material parameters of the FGPM coating are assumed to have the exponential function form along the thickness direction. The contact region is composed of the stick region and slip region where Coulomb friction law with a constant friction coefficient is assumed. The Hankel integral transform technique is used to formulate and convert the contact problem with a finite friction to a system of coupled singular integral equations. The approximate method developed by Goodman (1962) is applied to solve the coupled integral equations. The effects of the gradient index, friction coefficient and different piezoelectric substrates on the normal and radial tractions are investigated and discussed by using typical numerical examples. The results presented in this paper may provide us some useful guidelines to better understand the contact behavior and find an efficient way to improve the resistance of the coating to the contact damage.

2. Derivation of the integral equations

A rigid spherical indenter, which is a perfect electric insulator, is pressed into a functionally graded piezoelectric material (FGPM) coating of thickness h_0 bonded to a piezoelectric substrate as a half-space (Fig. 1). The force P induces a contact region which consists of an inner stick region ($r \le b$) and an outer slip region ($b \le r \le a$) where the Coulomb's law of friction with a constant friction coefficient f is applied. The poling direction of the FGPM coating-substrate system is parallel to the z-axis of the cylindrical coordinate system (r,θ,z). The variations of the material constants in the FGPM coating can be described in the following exponential forms:

$$\{c_{kl}(z), e_{kl}(z), \in_{kl}(z)\} = \{c_{kl0}, e_{kl0}, \in_{kk0}\}e^{\beta z}, \quad 0 \le z \le h, \tag{1}$$

in which c_{kl},e_{kl} and $\in_{kl}$ are the elastic, piezoelectric, and dielectric constants, respectively. Here, the index β denotes a material graded parameter of the coating, c_{kl0}, e_{kl0} and $\in_{kl0}$ are the material properties of the piezoelectric substrate.

The equilibrium equations for the FGPM coating can be represented as (Ueda, 2010)

$$\begin{aligned} &c_{110}\left(\frac{\partial^2 u_{rj}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{rj}}{\partial r} - \frac{u_{rj}}{r^2}\right) + c_{440}\frac{\partial^2 u_{rj}}{\partial z^2} + (c_{130} + c_{440})\frac{\partial^2 u_{zj}}{\partial r \partial z} + (e_{310} + e_{150})\frac{\partial^2 \phi_j}{\partial r \partial z} \\ &+\beta\left\{c_{440}\left(\frac{\partial u_{rj}}{\partial z} + \frac{\partial u_{zj}}{\partial r}\right) + e_{150}\frac{\partial \phi_j}{\partial r}\right\} = 0, \end{aligned} \tag{2a}$$