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Prediction method for permeability of porous media with tortuosity effect based on an intermingled fractal units model

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1. Introduction

The porous media theory has widely been applied to describe the mechanical behaviors of various materials such as soils, rocks, wood, bones, or industrial foams (Nguyen, Rohan, & Naili, 2016). But the microstructure is too complicated to get analytic solution of inner flow without simplifying and permeability predictions from such flow simulations are controlled by parameters whose settings must be calibrated in order to produce realistic modeling results (Hosa, Curtis, & Wood, 2016). Commonly used mathematical models cannot realistically reflect the permeability of complex porous media because of their empirical constants. In the 1967, the French mathematician Mandelbrot first proposed the concept of fractal and then created the fractal geometry theory (Mandelbrot,1983) that is an area of mathematics under construction. Although fractals receive considerable current attention, they are a newcomer to the history of geometry and to the task of describing physical objects (Barnsley & Vince, 2013). For example, even allometry relations (ARs) in physiology are nearly two hundred years old, however, the detailed application of fractal geometry to the explanation of intraspecies ARs is a little over a decade old (West & West, 2012). Fractals are objects which have similar appearances when viewed at different scales, such objects have details at arbitrarily small scales, making them too complex to be represented by Euclidian space (Rakhshandehroo, Shaghaghian, Keshavarzi, & Talebbeydokhti, 2009). There are many natural phenomena such as topographic surface, jagged shorelines, fickle changing clouds and seismic activities. For instance, based on the global gridded data generated from the Lunar Reconnaissance Orbiter, Cao, Cai, and Tang (2015) carried out their fractal measure to interpret lunar fractures by using qualitative (similar ratio) and quantitative (fractal dimension) approaches of fractal geometry. Salman and Truskinovsky (2012) provided evidence that a two-dimensional model is already adequate for describing power law statistics of avalanches and fractal character of dislocation patterning. Much in the same way, many literatures have suggested that the interspaces in real porous media have fractal characteristics in a certain range of scales from micrometer to nanometer (Lei, Dong, Mo, Gai, & Wu, 2015). Jacquin and Adler (1987) analyzed some geological structures, measured the spreading dimension for some of them and presented the procedure on the analysis of transport processes in porous media. Aniszewska (2015) applied fractal geometry to examine defects growth in porous ceramic specimens and the entire structure of defects enclosed

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in macroscopic volume is treated as a single fractal. Meanwhile, different fractal approaches have been taken to characterize fluid flow behaviors and predict the permeability of porous media, which have attracted much attention during the past several decades. Othman, Helwani, and Martunus (2010) developed the fractal permeability model for bi-dispersed porous media based on the fractal characteristics of pores in the membrane. Sheng, Li, Tian, Huang, and Chen (2016) have shown that the gas permeability is an integral value of individual permeabilities contributed from pores of different scales, they used fractal theory to build a shale-gas permeability model, particularly considering the effects of multiscale flow within a multiscale pore space. Ge, Fan, Deng, Han, and Liu (2016) presented an improved fractal model for pore structure evaluation and permeability estimation based on the high-pressure mercury porosimetry data, the fractal dimension distributions and relevant parameters are used to characterize the pore structure and permeability. Despite progress, or perhaps because of the microstructure is too complicated to get permeability prediction without simplifying the flow paths as straight streamlines, using fractal geometry to quantify the tortuosity effect on permeability of porous media has been limited (Behrang & Kantzas, 2017; Daigle, 2016; Pia & Sanna, 2014; Tan, Liu, Li, Zhang, & Cai, 2015). However, tortuosity affects the permeability of porous media that is often used as an adjustable parameter in models of transfer properties through porous media. This parameter, not reducible to classical measured microstructural parameters like specific surface area, porosity, or pore size distribution, reflects the efficiency of percolation paths, which is linked to the topology of the material (Barrande, Bouchet, & Denoyel, 2007). Hantel, Armstrong, DaRosa, and l'Abee (2017) considered tortuosity cannot be assessed directly and characterized it in polyetherimide membranes based on gurley and electrochemical impedance spectroscopy. Laudone, Gribble, Jones, Collier, and Matthews (2015) measured the electric conductivity of inter-pore brine to determine experimental tortuosity for two porous limestones and one porous sandstone, the above method is particularly useful for the many materials that can be characterized by mercury porosimetry or porometry, but for which tortuosity cannot be measured directly. Sevostianova, Leinauer, and Sevostianov (2010) evaluated the tortuosity from measurements of the bulk electrical conductivity of saturated soils at different levels of electrolyte content and different conductivities of the electrolyte.

The main goal of the paper is to put forward a new method to solve the tortuosity problem of porous media for permeability prediction by using the fractal geometry. The focus is two-fold. First, to emphasize a new practice of comparing only the pore size distribution between the predictions and the experimental ones, adding to older publications concentrating on replicating the pore size distribution and the volume fraction of voids experimental. Second, to concentrate on the core of the subject—using fractal expressions for predicting permeability of porous media, addressing the points of tortuosity effect. Although some notions are at the historical foundation of the subject, new practice and exciting fractal expressions have recently come to light. Section 2 provides a general procedure that can be followed to predict permeability of porous media with tortuosity effect by using the fractal geometry, a generalization of the concept for an intermingled fractal units model is also included. Some calculation methods such as of tortuosity and permeability, are introduced informally in this section, but are defined formally in the later section. In Section 3, followed by microscopic pore structure stating that it is reasonable to define characteristics of porous media in context of fractal theory. The section of formula derivation plays a key role in the construction of fractal expressions for calculating permeability with tortuosity effect. The comparison between the experimental data and the calculation results is given in Section 4. This leads to the demonstration of the accuracy and reliability of the proposed method and what it implies regarding the engineering practice. Summary and conclusions are given in Section 5.

2. Prediction method

Natural porous media are physically not easy to study, they are usually schematized by means of a specific deterministic model, like the Menger's Sponge (Mandelbrot, 1977; Vita, De Bartolo, Fallico, & Veltri, 2012). But the adoption of "simple models", the most noted of which is effectively exemplified by Menger sponge presents two problems: the succession of the pore sizes which count only a few terms for every decade and the distribution of the volumes concentrated towards the class of larger pore diameters (Atzeni, Pia, & Sanna, 2008). An intermingled fractal units model (denominated IFU) is developed by varying some constructive aspects of the Sierpinski carpet, which can be used effectively to describe very common structures which present one or more peaks in their pore size distributions (Pia & Sanna, 2013), including porous media of course.

Objectively, we can translate a set of microstructural shapes in an easily describable way to obtain a number of elementary units of the Sierpinski carpet type with different scales so that each fractal unit has its own fractal dimension, iteration times and a set of radii. By comparing more intermingled fractal units, the practice shows that the predicted pore volume fraction of voids can always present the best fit with available experimental data after a certain number of iterations. Therefore, omit a contrast of them and compare only pore size distribution between the predictions and the experimental ones can ensure that IFU model is able to simulate the geometry of the microstructure effectively. In general, the tortuosity τ is defined as the ratio of an effective path length (throughout the porous medium) to the shortest path length (length of the porous medium) (Carman, 1939; Letellier, Fierro, Pizzi, & Celzard, 2014) (see Fig. 1). Consider the problems of the tortuosity in porous media often means that the difficulty in permeability calculation is obviously increased, since the fractal dimension of the deviousness, D_t shows up as an exponential form in the formula for calculating the tortuosity length, L_t of the capillaries (Pia & Sanna, 2014). However, fluid flow in porous media are directly influenced by the tortuosity of the transport paths in the pore space (Ghanbarian, Hunt, Sahimi, Ewing, & Skinner, 2013). To deal with this problem, a geometry model for tortuosity of streamlines in three-dimensional porous media is introduced and expressed as a function of