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Peridynamic formulations enriched with bond rotation effects

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ABSTRACT

Limited by its fixed effective Poisson's ratio, the bond-based central force peridynamic theory fails to properly describe shear deformation of solids. In this paper, the classical peridynamic theory is enriched with bond rotation effect and reformulated in the thermodynamic framework with emphasis on mathematical derivation of the micro elastic moduli. The macroscopic strain energy of the peridynamic system is completed by involving the local shear stain for both 3D and 2D problems. In each case, the effective elasticity tensor is derived rationally and compared to the results of continuum solid mechanics, which allows relating the micro stiffness parameters to the common macro elastic constants. For validation, numerical tests are performed to assess the prediction of Poisson's ratio. It is shown that the enriched peridynamic model is capable of removing to a large extent the limitation of Poisson's ratio and consistent from the viewpoint of strain homogenization.

1. Introduction

The nonlocal peridynamics, viewed as an alternative theory of solid mechanics (Silling, 2000), is attracting a dramatically increasing interest in modelling elasticity and elastic damage of quasi brittle solids (Huang et al., 2015). The classical bond-based peridynamic model formulates mechanical problems in the form of integral equations rather than partial differential equations, making it more suitable for dealing with both continuous and discontinuous problems than the traditional finite element method. In peridynamic theory, it is assumed that two material particles will interact once they are located in a same material horizon and the pairwise central force between them is transmitted via a bond. The Silling's bond-based peridynamic model involves a single micromodulus to describe the micro elastic bond stretch response. It has been applied successfully to model bulk expansion behaviour by means of bond stretch, but fails to predict correctly shear deformation process. In linear isotropic case, because the bond stretch stiffness parameter is dependent only on the bulk modulus, the effective Poisson's ratio ν is proved to always be 1/3 under plane stress conditions and 1/4 under both 3D and plane strain conditions. As a result, even the linear elasticity of isotropic solids cannot be fully described by the classical peridynamic theory.

To overcome this shortcoming, various efforts have been made. Silling, Epton, Weckner, Xu, and Askari (2007) further proposed the state-based peridynamic theory, which allows to remove the limitation of Poisson's ratio but makes computation process more complex in view that the interaction forces are dependent on the deformation state of all family members of material particles. Sarego, Le, Bobaru, Zaccariotto, and Galvanetto (2016) showed that the Poisson's ratio in the state based peridynamic model can vary in the range from 0.1 to 0.45 for 2D problems. Gerstle, Sau, and Silling (2007) proposed a

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micropolar peridynamic model by adding to the classical formulations pairwise peridynamic moments. It is noted that in the micropolar peridynamic model, each bond was viewed as a micro cantilever beam with two stiffness parameters, one related to stretch deformation and the other to bending effect. Chowdhury, Rahaman, Roy, and Sundaram (2015) proposed a state-based micropolar peridynamic theory by introducing additional micro-rotational degrees of freedom to each material point of linear elastic solids. For plain stress problems, Gerstle et al. (2007) succeeded in formulating the two microparameters in terms of the macro elastic constants (for instance, the Young's modulus and the Poisson's ration). Recently, Prakash and Seidel (2015) developed a two-parameter linear elastic bond-based peridynamic model, where two springs were introduced to describe the displacements in the normal and tangential directions, respectively, and the peridynamic macroelastic strain energy is determined under plain stress conditions. By considering an isotropic homogeneous body with a homogeneous bi-axial strain field, Prakash and Seidel (2015) further set up the relationships between the two microparameters and the conventional macro elastic constants. It is, however, found that in the particular case where $\nu = 1/3$, the normal micro parameter obtained by Prakash and Seidel (2015) cannot reduce to the known result reported by Silling and Askari (2005) for bulk expansion tests. Recently, Ren, Zhuang, and Rabczuk (2016) proposed a new peridynamic formulation with shear deformation for linear elastic solid by subtracting the rigid body rotation part from the total deformation where the limitation of Poisson's ratio was not specifically handled, however.

It is worthy noticing that the restriction on Poisson's ratio in numerical simulation is a well known theoretical issue for bond-based models, lattice spring models, as well as particle based contact models. For example, in the virtual internal bond (VIB) model proposed by Gao and Klein (1998), the continuum is simulated as a network of randomized material particles connected via virtual bond. It is also shown that the effective Poisson's ratio in the VIB model is fixed at 1/4 for plane strain problems and 1/3 for plane stress problems. Zhang and Ge (2006) made an useful improvement to the VIB framework by introducing bond rotation and relating the local stiffness parameters to the elastic constants of isotropic continua. In the peridynamic context, a non-ordinary, state-based model was proposed by O'Grady and Foster (2014) based on the concept of a rotational spring between bonds. It is also noted that in the discrete element method using a grillage of structural elements (Griffiths & Mustoe, 2001), the micro stiffness parameters present the same limitation of Poisson's ratio.

Up to now, various extensions have been made for divers application purposes of the peridynamic method. However, the basic theoretical issue concerning the limitation of Poisson's ratio has not been thoroughly solved (Hu & Madenci, 2015; Zhou & Shou, 2016) and the relevant theoretical consistency from the viewpoint of strain homogenization has never been studied. In this paper, the classical bond-based central force peridynamic theory is extended by incorporating bond rotation effect with an additional micro elastic modulus involved. The macroscopic strain energy of peridynamic system is completed and reformulated by introducing local shear strain for both the 3D and 2D problems. For each case, the effective elasticity tensor is derived in a rational and concise way. Their comparisons with the results from classic continuum solid mechanics allow to relate explicitly the micro stiffness parameters to the common macro elastic constants. Differing from the previous works, the mathematical derivations are performed in a unified framework, and mathematically consistent in view that the micromoduli we obtained can reduce to the result from a bulk expansion test ($\nu = 1/4$) and that the requirement on strain homogenization is satisfied. As a first stage of validation, numerical tests are performed to assess the prediction of Poisson's ratio by the modified peridynamic model.

Throughout the paper, the following notation on tensorial products of any two second-order tensors $\boldsymbol{A}$ and $\boldsymbol{B}$ will be used: $(\boldsymbol{A}\otimes\boldsymbol{B})_{ijkl} = A_{ij}B_{kl}$ and $(\boldsymbol{A}\otimes^s\boldsymbol{B})_{ijkl} = (A_{ik}B_{jl} + A_{il}B_{jk})/2$. Similarly, the tensorial product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is denoted by $(\boldsymbol{a}\otimes\boldsymbol{b})_{ij} = a_ib_j$ and its symmetric part is taken as $(\boldsymbol{a}\otimes^s\boldsymbol{b})_{ij} = (a_ib_j + a_jb_i)/2$.

2. The classical bond-based peridynamic model

This part summarizes the main constitutive equations of the classical peridynamic theory set up by Silling (2000). For later comparisons, the local normal elastic modulus is determined.

2.1. Constitutive equations

The classical bond-based peridynamics is actually a nonlocal particle-based numerical method in which solid body is discretized by a limit number of material particles. Let us consider two particles $\boldsymbol{x}$ and $\boldsymbol{x}'$ in the initial configuration of a solid region R (see Fig. 1), whose relative position vector is given by

$$\boldsymbol{\xi} = \boldsymbol{x}' - \boldsymbol{x} \tag{1}$$

Assume that at instant t the two material particles are displaced respectively by $\boldsymbol{u}(\boldsymbol{x}, t)$ and $\boldsymbol{u}(\boldsymbol{x}', t)$, leading to the relative displacement vector

$$\boldsymbol{\eta} = \boldsymbol{u}\left(\boldsymbol{x}', t\right) - \boldsymbol{u}(\boldsymbol{x}, t) \tag{2}$$

Thus, the relative position of the two particles in the deformed configuration can be characterized by $\boldsymbol{\eta} + \boldsymbol{\xi}$. In addition, the following scalar variable is defined to describe the longitudinal deformation (stretch) of the bond (Silling & Askari, 2005)

$$\ell\left(\boldsymbol{\eta}, \boldsymbol{\xi}\right) = \frac{\|\boldsymbol{\eta} + \boldsymbol{\xi}\| - \|\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|} \tag{3}$$