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Multiscale analysis of non-periodic stress state in composites with periodic microstructure

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ABSTRACT

The size of a fine scale domain to be employed for accurate evaluation of a strongly non-periodic stress distribution in composites with periodic microstructure can be relatively large. Consequently, direct numerical simulation approach may become impractical.

A method suggested to overcome this difficulty hinges on the Discrete Fourier Transform. It allows to determine a non-periodic stress field in the fine scale domain that includes many periodic cells by analysis of a single representative (repetitive) cell with Bloch type boundary conditions. The analysis is carried out with respect to the complex-valued displacements and stress transforms, and actual values of the stress strain field components at any point of the fine scale domain are given by the inverse transformation. The method is illustrated by the analysis of crack problems for the two-dimensional periodically voided material.

1. Introduction

The computational cost of direct numerical simulation for problems where several length scales are involved led to intensive development of multiscale analysis methods (Belytschko & Song, 2010; Farhat & Roux, 1991; Fish & Yu, 2001; Guidault, Allix, Champaney, & Navarro, 2007; Ibrahimbegovic & Markovic, 2003; Ladeveze, Loiseau, & Dureisseix, 2001; Lloberas-Valls, Rixen, Simone, & Sluys, 2012). It is conventional to divide these methods into two main groups based on hierarchical and concurrent approaches (Belytschko & Song, 2010; Lloberas-Valls et al., 2012). For the former (called also sequential) method, micro- and macroscale considerations are separated and, after application of a homogenization technique on a microscale, the information is passed to a macroscale model. In concurrent methods, the entire domain is decomposed into a coarse scale and one or several fine scale subdomains (usually, fine scale domains are embedded into the coarse scale one, e.g., Belytschko & Song, 2010; Guidault et al., 2007), which are coupled and treated simultaneously. The coupling of fine subdomains and fine/coarse ones can be accomplished, for example, by means of Lagrange multipliers providing compatibility at corresponding interfaces (e.g., Farhat & Roux, 1991). It is also possible to formulate interface conditions by splitting unknown components of elastic fields into micro- and macrovariables, and require that only macro-quantities must obey the transmission relations (Ladeveze et al., 2001). The important point of a concurrent approach is that it usually allows to take advantage of modern computer architecture with parallel processors. In addition, there are a number of hybrid multiscale frameworks (e.g., Lloberas-Valls et al., 2012) employing features of hierarchical and concurrent methods.

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The problems related to the fracture phenomenon in heterogeneous materials represent an important subset of problems with several length scales. In this case in order to predict crack propagation observed on a macroscale, one has to know the precise microscale near-tip stress distribution. Consequently, there is no merit to employ hierarchical methods for these problems. On the other hand, application of concurrent methods may be hampered by a large number of degrees of freedom in the fine-scale domains, which are involved in the analysis.

It appears, that in the specific case of pure periodic heterogeneity, the volume of calculations in the analysis of the fine-scale domain can be reduced significantly by taking advantage of the translational symmetry of material microstructure. The multiscale strategy suggested for this case in the present paper hinges on the Discrete Fourier Transform. The goal of the work is to develop an efficient numerical method for the analysis of arbitrary loaded composite domain of arbitrary shape. Consequently, in spite of the periodicity in the geometry and elastic properties, the stress state in the domain is non-periodic and the boundary conditions for the cells, to which the domain may be separated, are non-periodic as well. The cells are identified by an index and this index serves as a discrete parameter in the Fourier transform.

The considered problem should not be confused with the problems treated by the use of the Discrete Fourier Transform where both composite microstructure and stress state are periodic. The Moulinec–Suquet method (see Moulinec & Suquet, 1998; 2003; Vinogradov & Milton, 2008) employs the DFT (in the form of Fast Fourier Transform (FFT) modification) for the analysis of local and overall responses of periodic composites. In this case a series expansion for the elastic fields and effective tensors is considered and FFT is employed for discretizing the field within the repetitive cell subjected to periodic boundary conditions.

Multiple references to the DFT-based non-periodic analytical solutions for unbounded periodic lattices can be found in the monograph of Slepyan (2002). In the field of numerical analysis, the DFT-based solution methods were developed for the analysis of arbitrary loaded structures (Karpov, Stephen, & Dorofeev, 2002; Ryvkin, Fuchs, & Nuller, 1999) as well as nano-scale atomic systems (Qian, Phadke, Karpov, & Liu, 2011). For the analysis of elastic continuum, DFT was intensively employed in the framework of the representative cell approach in conjunction with the finite elements method or the higher-order theory (Ryvkin & Aboudi, 2007; Ryvkin & Nuller, 1997). Examples of the implementation of this approach for the analysis of unbounded periodic domains with non-periodic stress state can be found in the review papers (Aboudi & Ryvkin, 2013; Ryvkin, 2008); unbounded periodically voided domains with semi-infinite cracks were considered in Ryvkin and Aboudi (2016) and Ryvkin and Hadar (2015). In the present paper a general approach applicable for an analysis of arbitrary bounded as well as unbounded periodic domains will be presented.

In general, the application of the DFT to a problem for an elastic system possessing translational symmetry in certain direction leads to a problem for a single period where the transforms of the unknown quantities are related by the specific Bloch-type boundary conditions

$$f_- = f_+ e^{i\varphi} \tag{1}$$

Here the subscripts "+" and "-" denote the opposite boundaries of the periodic cell, i is imaginary unit and φ is the transform parameter. Relations of this type are typical for the analysis of spatially periodic mechanical systems (Brillouin, 1953). El Hami and Radi (1996) obtained them on the basis of group theory. The goal of the present work is to demonstrate how it is possible to plug-in the DFT based Representative Cell Method (RCM) suggested in Ryvkin and Nuller (1997) into a general multiscale analysis of an arbitrary shaped body made of periodic material. The application of the DFT in the framework of the suggested approach stipulates the linearity of the governing equations, while possible ways to consider dynamic and non-linear problems are discussed in the conclusions section.

The paper is organized as follows. In the next section the general scheme for the multiscale analysis of an arbitrary loaded two-dimensional periodic domain is presented. In Section 3, the modifications are introduced allowing the analysis of periodic materials with an embedded crack, and in Section 4, the technique application is demonstrated by the analysis of the crack problems for 2D periodically voided material. The importance of the correct choice of the fine-scale domain size is demonstrated. In the final section, several concluding remarks are drawn.

2. Fine-scale analysis of periodic material

Consider a periodic elastic two-dimensional material occupying domain Ω (Fig. 1). The domain is subjected to a body force $\mathbf{P}_0$ and on the respective parts of its boundary Γ_Ω the displacement $\mathbf{u} = \mathbf{u}_0$ and traction $\mathbf{t} = \mathbf{t}_0$ boundary conditions are given. The periodicity of the material microstructure is characterized by a length parameter $l_0 = max\{2l_1, 2l_2\}$, where $2l_1$ and $2l_2$ are the magnitudes of the vectors defining the translational symmetry of the material. Assume that the region of interest Ω_{int}, where the precise stress-strain field is to be evaluated, is located in the inner part of Ω, so that it is possible to consider this region as a part of a relatively large rectangular domain Ω_f (note, that Ω_f may be also a parallelogram, and rectangular shape is adopted for convenience). Consequently, if we denote the characteristic length parameters of Ω_{int} and Ω_f as L_{int} and L_f, respectively, the following relations hold

$$L_f \gg L_{int} \quad \text{and} \quad L_f \gg l_0 \,. \tag{2}$$

The rectangular domain Ω_f, $-L_i \le X_i \le L_i, i = 1, 2$ is viewed as an assemblage of repetitive cells (Fig. 2) and (their number N is expressed through domain dimensions and the dimensions of the representative periodic cell Ω^*, $2l_1 \times 2l_2$, as $N = N_1 N_2$