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Hydraulic fracture crack propagation in an elastic medium with varying fracture toughness

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A R T I C L E I N F O

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A B S T R A C T

Hydraulic fracture crack propagation in an isotropic elastic medium with varying fracture toughness is considered. The crack is subjected to the pressure of fluid injected at a point on the crack surface. The fluid is viscous Newtonian and incompressible, the medium is impermeable. The analysis of crack growth is based on the three-parameter model of pressure distribution on the crack surface. The model allows one to satisfy the condition at the point of fluid injection, the balance equation of the volume of injected fluid and the crack volume, and fracture criterion at the crack edge. The analysis of evolution of the crack boundary is based on an original method of fast numerical solution of crack problems. In this method, Gaussian approximating functions are used for discretization of the problem, and fast Fourier transform technique is applied for solution of the discretized equation. The method allows constructing the crack boundary at discrete time moments for media with varying fracture toughness and time dependent positive injection rate. Examples of hydraulic fracture crack propagation in the medium that consists of two half-spaces with different fracture toughnesses and a layer of the material with another fracture toughness in a homogeneous elastic medium are considered. Evolution of the crack boundaries in the process of fluid injection, time dependence of pressure distributions and crack openings are presented in graphic forms.

1. Introduction

A crack in an elastic medium subjected to fluid injection is the basic model for simulation of hydraulic fracture processes in rock materials (Fig. 1). This model has been the object of theoretical and experimental studies for about 60 years. The system of equations describing crack evolution in the process of hydraulic fracture was derived at the end of the last century and has been used by many authors (see, e.g., Adachi, Siebrits, Peirce, & Desroches, 2007; Savitski & Detournay, 2002). It was shown that the problem reduces to a system of non-linear integro-differential equations in the region with moving boundary. Analytical solutions of this system do not exist even in simplest cases, numerical solutions were presented in a number of works (e.g., Adachi et al., 2007; Meyer, 1989; Zhou & Hou, 2013, and others). Detailed surveys of various methods of solution of hydraulic fracture problems can be found in Berriman (2016) and Wrobel and Mishuris (2015).

As a rule, actual rocks are heterogeneous, and this fact should be taken into account in numerical simulations of hydraulic fracture. Since heterogeneities may result development of a non-planar crack with a complex boundary, numerical simulation of crack growth becomes a cumbersome computational problem. For numerical solution, the system of integro-differential equations has to be discretized with respect to time and spatial variables. A grid of nodes on the crack surface

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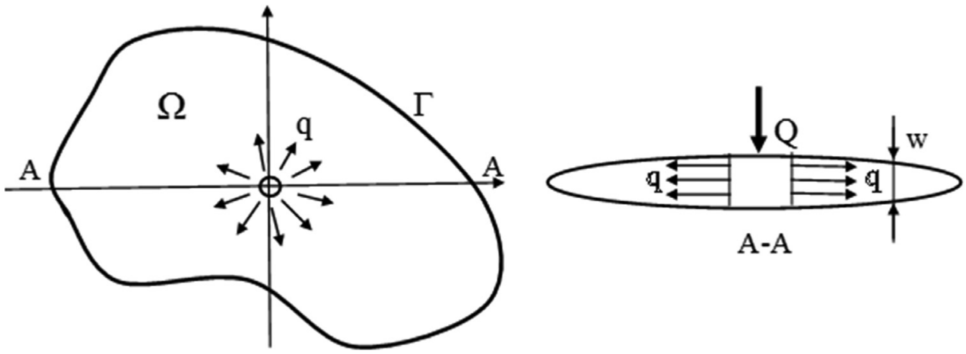

Fig. 1. A crack subjected to pressure of fluid injected at a point of its surface.

and in the surrounding medium must be fine enough to calculate crack opening and stress intensity factors (SIFs) at the crack edges with sufficient accuracy. Since the crack boundary moves, the node grid should be adjusted at each time step of the propagation. In addition, at a fixed time moment, the solution needs several iterations in order to find the crack boundary that corresponds to the actual distribution of fracture toughness in the material. In some publications (Adachi et al., 2007; Zhou & Hou, 2013), numerical solutions of the problems of hydraulic fracture crack propagation in heterogeneous materials are given without revealing the details of the numerical algorithm and analysis of its accuracy.

An important characteristic of the hydraulic fracture process is the fluid pressure distribution on the crack surface. It was shown (see, e.g., Savitski & Detournay, 2002) that for hydraulic fracture process with continuously moving crack boundary, the pressure should have singularity at the crack tip. This result is in controversy with the fact that fluid cannot sustain large tension stresses. In Kanaun (2017) the discrete model of crack propagation was proposed. It was assumed that the actual process of crack growth is a series of small discrete steps, and each step consists of three stages: growth of the crack volume by a fixed crack boundary, an instant crack jump to a new boundary defined by fracture criterion, and filling the new crack configuration by fluid. In this model, singular stresses in fluid at the crack tip do not appear. In the case of a homogeneous medium the model gives reasonable results that are close to the results of other authors. It is also shown that for small fluid viscosity, the pressure distribution can be described by a simple three-parameter model.

In this work, hydraulic fracture crack propagation in an isotropic elastic medium with constant elastic moduli but varying in space fracture toughness is considered. It is assumed that the crack is planar, and fracture criterion $K_I(x) = K_{Ic}(x)$ is satisfied at points of the crack boundary in the process of crack growth. Here $K_I(x)$ is the SIF for a crack subjected to fluid pressure and $K_{Ic}(x)$ is fracture toughness at point x. For numerical simulation of crack propagation, a three-parameter model of pressure distribution on the crack surface is used. In this model, the pressure is approximated by the sum of logarithmic and constant terms with time depending coefficients. The model allows one to satisfy the balance of the volume of the injected fluid and the crack volume, the condition at the point of fluid injection, and the fracture criterion on the crack boundary. For construction of the crack boundary in the process of crack growth, an efficient numerical method proposed by Kanaun (2007) is used. The method is based on application of the Gaussian approximating functions for discretization of the integral equation of the crack problem. The theoretical background was developed in Maz'ya and Schmidt (2007). Gaussian functions allow constructing the elements of the matrix of the discretized problem in explicit analytical forms. Using the fast Fourier transform technique for iterative solution of the dicretized problem accelerates calculation substantially. The special cases of a homogeneous medium, of the medium that consists of two half spaces with different fracture toughnesses, and of the medium containing a layer of the material with different fracture toughnesses are considered.

2. Three-parameter model of pressure distribution for a planar crack subjected to fluid injection

Consider an infinite isotropic elastic medium containing an isolated planar crack of arbitrary shape. The crack is subjected to pressure $p(x, t)$ caused by fluid injected at point x^0 of the crack surface (Fig. 1). The fluid injection rate $Q(t)$ is a given function of time t, x is a point on the crack surface Ω. The point x^0 is taken as the origins of polar (r, φ) and Cartesian (x_1, x_2) coordinate systems in the crack plane. Fluid pressure $p(x, t)$ produces crack opening vector orthogonal to the crack surface: $w(x,t)\mathbf{n}(x)$. For an arbitrary finite pressure distribution, crack opening $w(x,t)$ has the following asymptotics near the crack boundary Γ:

$$w(x,t) = \beta(x^b,t)\sqrt{s} + O(s^{3/2}), \quad x^b \in \Gamma. \tag{1}$$

Here s is the distance from point $x \in \Omega$ to point $x^b \in \Gamma$ along the normal vector $\boldsymbol{\gamma}$ to Γ in plane Ω. The crack volume $V(t)$ is

$$V(t) = \int_{\Omega(t)} w(x,t)d\Omega. \tag{2}$$