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Nonlinear bending and forced vibrations of axially functionally graded tapered microbeams

Mergen H. Ghayesh[a,*], Hamed Farokhi[b], Alireza Gholipour[a], Mohammad Tavallaeinejad[b]

[a] School of Mechanical Engineering, University of Adelaide, South Australia 5005, Australia
[b] Department of Mechanical Engineering, McGill University, Montreal, Quebec, Canada H3A 0C3

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ABSTRACT

The forced nonlinear size-dependent vibrations and bending of axially functionally graded (AFG) tapered microbeams are examined incorporating extensibility. Employing the modified version of the couple stress-based theory, the nonlinear partial differential equations for the transverse and longitudinal motions for a clamped-clamped AFG tapered microbeam are obtained via Hamilton's principle. The variation of the mechanical properties and the cross-section of the AFG microbeam along the length are included in the equations of motion based on exponential distributions of the moduli of elasticity, mass density, Poisson's ratio, and cross-sectional area. The Galerkin method is utilised to obtain a set of discretised nonlinear differential equations of ordinary type; this set of equation is solved with the help of Houbolt's finite difference technique together with the Newton-Raphson method. The effects of the small-scale parameter, the gradient index, material properties variation, and the taper ratio on the nonlinear vibrations of the microsystem are investigated.

1. Introduction

With the rising demands in modern microtechnologies, the conventional uniform and homogeneous materials are no longer able to efficiently meet the emerging needs in microelectromechanical systems (MEMS) industry; hence, new advanced classes of materials with special mechanical properties are being fabricated. Microbeams of functionally graded (FG) materials and laminated composites are examples of these advanced materials which have attracted a substantial attention from engineers and researchers due to their unique properties such as high strength, toughness, thermal resistance, and low density (Huang, 2008; Lü, Lim, & Chen, 2009; Witvrouw & Mehta, 2005). Mechanical properties of functionally graded (FG) materials vary gradually along a desired direction (Li & Hu, 2016); as opposed to conventional FG microbeams whose mechanical characteristics vary along their thickness, the material properties of axially functionally graded (AFG) microbeams vary along the longitudinal axis (i.e. along the length) of the microbeam. In addition to mechanical properties, the tapered microbeams possess varying cross-sectional area; thus, the governing differential equations of AFG tapered beams are partial differential equations with variable coefficients – obtaining the solutions for such a microbeam with non-uniform distribution of material properties and complex geometry is very challenging. Hence, application of well-optimised

* Corresponding author.
E-mail address: mergen.ghayesh@adelaide.edu.au (M.H. Ghayesh).

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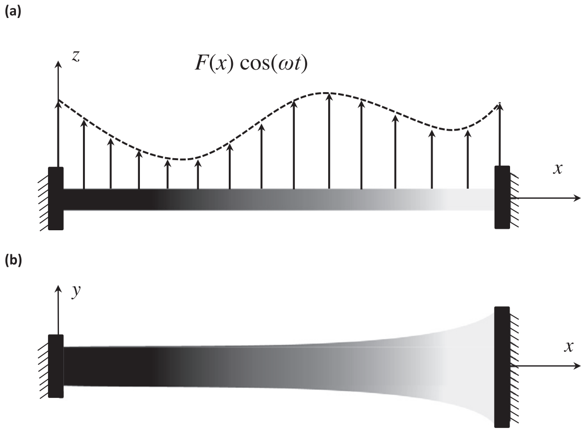

Fig. 1. Schematic representation of an extensible axially functionally graded (AFG) tapered microbeam subject to a distributed harmonic excitation load in the transverse direction.

numerical techniques is essential. An important issue in the deformation behaviour of microbeams is their inherent size-dependence (Akgöz & Civalek, 2013a, 2011; Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani & Bahrami, 2016; Dehrouyeh-Semnani, BehboodiJouybari, & Dehrouyeh, 2016; Farokhi, Ghayesh, Gholipour, & Hussain, 2017; Ghayesh, Amabili, & Farokhi, 2013a, Hosseini & Bahaadini, 2016; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008; Mojahedi, 2017; Mojahedi & Rahaeifard, 2016; Rahaeifard, 2016; Shafiei et al., 2016a, Shafiei, Kazemi, & Ghadiri, 2016b, Taati, 2016).

Most of the studies on FG microbeams are devoted to FG microbeams with material properties varying along the thickness (i.e. in the *transverse* direction); there are few studies in the literature which analysed the vibrations of *axially* functionally graded (AFG) microbeams. Akgöz and Civalek (2013b) examined the linear vibrations of non-uniform non-homogenous microbeams made of an AFG material using the modified couple stress theory (MCST) which aimed at calculating natural frequencies and mode functions. Another linear study on the vibrations of AFG microbeams, but this time using the Timoshenko theory, is done by Shafiei, Kazemi, and Ghadiri (2016c), who examined the free transverse, axial, and rotational vibrations of a rotating AFG microbeam using a single-mode approximation for each. An extension to a nonlinear model for AFG microbeams was performed by Simsek (Şimşek, 2015), who employed a single-mode discretisation for the free transverse vibrations; he employed MCST for incorporating micro-scale effects. There is also another paper in the literature by Shafiei et al. (2016b), who analysed the transverse and axial free vibrations of an AFG tapered microbeam via a single-mode truncation.

This paper investigates, for the first time, the coupled forced nonlinear vibrations as well as bending of an Euler-Bernoulli AFG tapered microbeam incorporating extensibility and size-dependence. MCST is employed in order to capture the size-dependent behaviour of the microsystem. Nonlinear distribution of the material and structural properties due to the variations of the width and material of the microbeam along its length are taken into account. The kinetic and potential energies of the microsystem are formulated and the governing equations of motion in the transverse and longitudinal directions are obtained with aid of Hamilton's principle. The Galerkin scheme is employed to obtain a set of discretised second-order coupled nonlinear equations, while retaining a large number of modes. Numerical simulations are performed utilising Houbolt's finite difference method together with the Newton-Raphson technique so as to study the effect of key parameters such as the gradient index and the small-scale parameter on the nonlinear oscillations of the AFG microsystem. Moreover, three different cases of taper ratios are considered via changing the width profile of the axially FG microbeam.

2. Equations of motion for AFG tapered microbeam and method of solution

Fig. 1 shows an AFG tapered microbeam of length L and thickness h with its left-hand end made of a metal material and the right-hand end made of a ceramic material. The microbeam is subjected to a harmonic distributed excitation $F(x)\cos(\omega t)$ with ω being the excitation frequency and $F(x)$ being the excitation amplitude at its centreline (where no rotations are imposed to the microsystem); t represents time. The transverse and longitudinal motions are denoted by $w(x, t)$ and $u(x, t)$, respectively.

The tapered microbeam is such that the variations of the material properties, namely elastic moduli E and μ, the mass density ρ, Poisson's ratio ν, material length scale l, as well as its geometrical properties, namely the cross-sectional area A