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Nonlinear stability of modulated Horton–Rogers–Lapwood problem

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ABSTRACT

Nonlinear stability theory is used to investigate free convection arising in a particular class of Horton–Rogers–Lapwood (HRL) problem. Time periodic modulation is imposed on either the temperature at the bounding surfaces or the gravitational field permeating the medium. The Brinkman model and the Boussinesq approximation govern the fluid flow. The energy formulation is followed and the Galerkin method is used to determine the relevant threshold for arbitrary values of the modulational amplitude and frequency. In general it is found that an increase in the modulational amplitude encourages convection ensuing at the threshold. The existence of a subcritical region is predicted and the importance of the nonlinear theory is established.

1. Introduction

Natural convection in a horizontal porous layer heated from below is one of the classical problems in the field of heat transfer. Horton and Rogers (1945) and Lapwood (1948) independently found that convection arises in an unbounded horizontal porous layer uniformly heated from below when the Darcy–Rayleigh exceeds $4\pi^2$. After their pioneering work, the HRL problem attracted several researchers who have taken into account several other additional constraints. Among them, parametric modulation is an important one due to its wide range of applications in engineering fields like growth of biological tissues, storage of agricultural products, geothermal extraction, electronic equipment designing and solidification. In particular in the crystal growth process, when a crystal is made to grow on the earth surface, gravity pulls the seed and grown crystals in the solution down whereas the fluid near the growing crystals is driven by buoyancy force, thereby affecting the lattice formation. These unfavourable sedimentary and buoyancy effects always exist in the terrestrial environment and these could be suppressed under microgravity conditions. For instance the experiment of Zhu, Li, Rogers, Meyer, and Ottewill (1997) to produce crystals from glassy samples in large volume was successful within two weeks in space which earlier failed even after a year on the earth. This clearly demonstrates the importance of modulational control over convection.

Venezian (1969) was the first to examine surface temperature modulation effect on the Benard problem for stress free boundaries. The studies which were made in the following decade to analyse the stability in a thermally modulated horizontal fluid system had been well documented by Davis (1976). The study concerning convection onset was extended to a thermally modulated densely packed porous system heated from below by Caltagirone (1976) using the Galerkin method and by Chhuon and Caltagirone (1979) theoretically, using the Floquet analysis and experimentally. Subsequently,

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Roppo, Davis, and Rosenblat (1984) carried out a weakly nonlinear stability analysis in a horizontal fluid layer subjected to thermal modulation and found the formation of hexagonal cells in the subcritical regime. Rudraiah, Radhadevi, and Kaloni (1990) determined convective instability region for a thermally modulated porous layer saturated by a viscoelastic fluid using perturbation method. They concluded that the elastic parameters and high values of the frequency delay the onset of convection whereas low values of the frequency advance it. Later Malashetty and Wadi (1999) used the perturbation method to determine the onset of convection in a sparsely packed porous system with stress free and thermally modulated boundaries. This was further analysed by Bhadauria (2007) for physical rigid boundaries using the Galerkin method. More recently, Singh, Hines, and Iliescu (2013) examined the effect of nonuniform thermal gradient on the stability of a ferrofluid saturated horizontal porous medium bounded by stress free and rigid boundaries and observed the existence of a subcritical region.

On the other hand Gresho and Sani (1970) and Gershuni, Zhukhovitskii, and Iurkov (1970) started investigating the effect of modulation in gravitational field on the Benard problem. Following their linear analysis, Clever, Schubert, and Busse (1993) made a nonlinear analysis and obtained the corresponding stability limits to a much wider region of parameter space. Yang (1997) then analysed the same problem and obtained instability limits in the case of a viscoelastic fluid. The effect of gravity modulation on the stability of a sparsely packed porous system heated from below was determined by Malashetty and Padmavathi (1997) using perturbation method for stress free boundaries. Govender (2004) analysed the effect of low amplitude *g-jitter* on the stability of a convective flow in a densely packed porous media heated from below under rigid boundaries. Later Boulal, Aniss, and Belhaq (2007) analysed the stability of a horizontal fluid layer heated either from below or from above subjected to quasiperiodic gravity modulation. Strong (2008) considered gravity modulation effect with arbitrary amplitude and frequencies using continued fractions. Her analysis was extended by Saravanan and Sivakumar (2010) for the Brinkman model using Hill's determinant and continued fraction methods. They found that the increase in the Darcy number delays the onset of convection and the stability region is wider for rigid boundaries compared to stress free ones.

The above discussion gives a brief review of the works concerned with the stability of thermally / gravity modulated horizontal fluid or fluid filled porous media. Nevertheless a complete review of the relevant literature shows the absence of a nonlinear study which provides sufficient condition for stability corresponding to a modulated fluid filled porous medium. Hence an attempt is made here to accomplish this in a porous medium of more general nature. At this stage one should notice that the nonlinear study made by Homsy (1974) is for a fluid layer. Most of the earlier studies in porous media have used the classical Darcy model for momentum balance which is valid only for flow through regular structures over the whole spectrum of the porosity. However this model is silent about the flow structure near the bounding surfaces where close packing of the porous material is not possible. We shall employ the Brinkman model which has a Laplacian term analogous to that appearing in the Navier–Stokes equations. This can take care of the boundary effects. According to Rajagopal (2007) these models can be obtained systematically by making severe approximations on a system of more general constitutive equations which are based on the mixture theory and can determine the dynamic behaviour of the solid matrix and the fluid filling the matrix. These equations are also introduced in detail by Straughan (2008). In the present work we shall take into account the nature of the boundaries while finding the nonlinear limits.

2. Mathematical formulation

We consider a porous medium of infinite horizontal extent confined between the surfaces $z = 0$ and $z = d$. Let the medium be homogeneous, isotropic, fluid saturated and exposed to gravity acting downwards opposite to the z-direction. The system is heated from below and is subjected to time periodic thermal modulation at the bounding surfaces and vertical gravity modulation. The Brinkman model is used to govern the flow through the medium and the Boussinesq approximation is assumed to take care of the density variation. The bounding surfaces are considered to be impermeable and are either tangential stress free or rigid.

The flow of a viscous and incompressible fluid through the medium in the above system is governed by

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{q} - \frac{\nu}{k}\vec{q} + \alpha T(1+\epsilon\ \cos(\Omega t))g_0\vec{k} \tag{1}$$

$$\nabla\cdot\vec{q} = 0 \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \kappa\nabla^2 T \tag{3}$$

subjected to

$$w = \frac{\partial w}{\partial z} = 0 \qquad \text{at } z = 0, d \quad \text{for rigid case} \tag{4}$$

$$w = \frac{\partial^2 w}{\partial z^2} = 0 \qquad \text{at } z = 0, d \quad \text{for free case} \tag{5}$$