International Journal of Engineering Science 120 (2017) 100–118

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Geometric analysis of hyper-stresses

Reuven Segev

Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel

ARTICLE INFO

Article history:
Received 1 November 2016
Accepted 14 July 2017

MSC:
74A10
53Z05
58A32

Keywords:
Continuum mechanics
High order stresses stress
Virtual power
Differentiable manifolds
Jet bundles
Iterated jets
Non-holonomic sections

ABSTRACT

A geometric analysis of high order stresses in continuum mechanics is presented. Virtual velocity fields take their values in a vector bundle W over the n-dimensional space manifold. A stress field of order k is represented mathematically by an n-form valued in the dual of the vector bundle of k-jets of W. While only limited analysis can be performed on high order stresses as such, they may be represented by non-holonomic hyper-stresses, n-forms valued in the duals of iterated jet bundles. For non-holonomic hyper-stresses, the analysis that applies to first order stresses may be iterated. In order to determine a unique value for the tangent surface stress field on the boundary of a body and the corresponding edge interactions, additional geometric structure should be specified, that of a vector field transversal to the boundary.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

The theory of hyper-stresses in continuum mechanics, *e.g.* Toupin (1962, 1964) and Mindlin (1964, 1965), accounts for phenomena not accounted for by the standard theory of stresses, such as edge interactions and surface tension. Although five decades have passed since this pioneering body of work has been published, various aspects of higher-order continuum mechanics are still under current research, *e.g.* dell'Isola, Seppecher and Madeo (2012), dell'Isola, Seppecher and della Corte (2015), Fosdick (2016), Mariano (2007), Münch and Neff (2016), Podio-Guidugli (2015). Notwithstanding the difficulties with higher order theories, in particular, those related to the specification of boundary conditions (*e.g.*, Bertram, 2017; Lam, F. Yang, Chong, J. Wang and P. Tong (2003)), these theories are motivated physically by the interaction of atoms with their next-to-nearest neighbors as has been observed in the early works mentioned above (see also Tarasov, 2015). Higher order theories also extend the variety of physical phenomena described by continuum mechanics (see for example Aifantis, 1992; Ru and Aifantis, 1993; Askes and Aifantis, 2011). These include, for example, surface tension and dependence on length-scale which is relevant in current microscopic devices and technologies.

This work is concerned with geometric analysis of smooth stresses of order k in continuum mechanics. In Segev (1986), for the setting where both the body B and space $\mathcal{S}$ objects of continuum mechanics are modeled as general differentiable manifolds, a hyper-stress theory was proposed in which the fundamental object is the configuration space Q containing all C^k-embeddings of the body into space. Using results on manifolds of mappings (*e.g.* Palais, 1968; Michor, 1980; Hirsch, 1976), it follows that the configuration space may be given the structure of a Banach manifold. The tangent space $T_\kappa Q$, at a

E-mail address: rsegev@bgu.ac.il

http://dx.doi.org/10.1016/j.ijengsci.2017.07.001
0020-7225/© 2017 Elsevier Ltd. All rights reserved.

generic configuration of the body $\kappa : B \to \mathcal{S}$, is interpreted physically as the space of virtual velocities. It may be identified with the space of C^k-sections, vector fields, of some vector bundle W, where the space of sections is equipped with the C^k-topology. A generalized force F of order k at the configuration κ is defined to be a continuous linear functional on the tangent space $T_\kappa Q$ and the value of the action of a force on a generalized velocity is interpreted as the corresponding virtual power.

It is shown there that forces may be represented by measures valued in the dual of the k-jet bundle, J^kW, of W. Locally, these measures are represented by a collection of tensors valued measures of orders 1 to k. These representing measures are referred to as variational (hyper-) stresses. The relation between a force system containing the forces of order k on all subbodies of B and a k-hyper-stress field, the analog of Cauchy's postulates, is studied in Segev (1986), Segev and DeBotton (1991) for the general case of stress fields that are as irregular as measures.

In the smooth case, the measures of the variational stress are represented by smooth sections S of the fiber bundle $L\big(J^kW, \bigwedge^n T^*B\big) = (J^kW)^* \otimes \bigwedge^n T^*B$ so that the value of the stress field at a point $x \in B$ is a linear mapping $(J^kW)_x \to \bigwedge^n T^*B$. Thus, the power expended by the force F for the generalized velocity w is given by

$$F(w) = \int_B S(j^k w), \tag{1.1}$$

where $S(j^kw)$ is the n-form whose value at $x \in B$ is $S(x)(j^kw(x))$, so that the integration above is well defined.

For the standard continuum mechanics case, $k = 1$, a procedure given in Segev (2002); 2013) and outlined in Section 3, makes is possible to write (1.1) in the form

$$F(w) = \int_B \mathbf{b}(w) + \int_{\partial B} \mathbf{t}(w), \tag{1.2}$$

where $\mathbf{b}$, the body force, is a section of $L\big(W, \bigwedge^n T^*B\big)$, satisfies

$$\operatorname{div} S + \mathbf{b} = 0, \quad \text{in } B, \tag{1.3}$$

and $\mathbf{t}$, the surface force, satisfies a generalization of Cauchy's formula

$$\rho \circ \sigma = \mathbf{t}, \quad \text{on } \partial B. \tag{1.4}$$

Here, σ, the traction stress, is a section of $L\big(W, \bigwedge^{n-1} T^*B\big)$ that generalizes the Cauchy stress, and ρ is the restriction of forms defined on TB to $T\partial B$. The traction stress is determined by the variational stress. It is emphasized that for the setting of general manifolds, two distinct objects represent the two functions of the classical stress object, namely, acting on derivative of velocities to produce power, and determining the surface force for various subbodies. The divergence operator for manifolds, as mentioned above and defined below, generalizes the standard divergence operator of second order tensors.

In this paper we study the geometric structure required to provide the analogous construction for smooth hyper-stresses of order k. In particular, we consider the geometric structure needed to determine the edge interactions induced by hyper-stresses using integral transformations in analogy with the analysis in dell'Isola et al. (2015, 2012). It is shown below that the setting of iterated jet bundles, $J^1(J^1W)$ for instance, is preferable to that of higher jet bundles, for instance, J^2W, respectively. Using iterated jet bundles makes it possible to apply the procedure for standard continuum mechanics, inductively.

Hyper-stresses in bodies induce tangent surface stresses on the corresponding boundaries. However, it is shown that on general differentiable manifolds, the induced surface stress, and hence the edge interactions, are not unique. For the unique determination of the tangent surface stress, one needs at least some specified vector field which is transversal to the boundary or an equivalent structure. The situation is similar to that described in Epstein and Tene (1973) and Epstein and de León (1998), where shell theory is considered. Evidently, for the case of a Riemannian manifold, the unit normal vector field provides such a transversal field naturally.

Section 2 introduces the relevant terminology and notation used for jet bundles associated with vector bundles. Section 3 reviews the relevant constructions of Segev (2002) regarding smooth stress distributions on manifolds as outlined above. Section 4 is concerned with hyper-stresses of order k, their representations and their invariant components. Some of the difficulties related to the analysis of hyper-stresses are indicated. Section 5 considers iterated jet bundles (see Saunders (1989)). Iterated jet bundles are of interest as their sections may have additional forms of incompatibility in comparison with sections of jet bundles. Forms valued in the duals of iterated jet bundles are referred to here as non-holonomic hyper-stresses. These are considered in Section 6. Due to the inductive nature of iterated jet bundles, it is sufficient to study the properties of the iterated jet bundle $J^1(J^1W)$. The vector bundle W itself may be a jet bundle, or an iterated jet bundle, of some other vector bundle. It is noted that every hyper-stress may be represented by non-holonomic hyper-stresses. The properties of non-holonomic hyper-stresses, in particular, the corresponding integral transformations associated with their action, are analyzed in this section for the case of general manifolds. Section 7 shows how the introduction of a particular vector field which is transversal to the boundary of a body induces a unique stress of a lower order on the boundary. Finally, in Section 8, the edge interactions induced by the non-holonomic hyper-stress are computed.

2. Notation and preliminaries

All manifolds considered here are viewed as chains or manifolds with corners so that we may use the Stokes theorem for integration of forms.

Download English Version:

https://daneshyari.com/en/article/5022641

Download Persian Version:

https://daneshyari.com/article/5022641

Daneshyari.com