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On longitudinal dynamics of nanorods

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ABSTRACT

The longitudinal dynamic problem of a size-dependent elasticity rod is formulated by utilizing an integral form of nonlocal strain gradient theory. The nonlocal strain gradient model accounts for the energies diffused from surrounding particles in a reference domain by utilizing the convolution integral over nonlocal kernel functions, and can account for micro/nano-structures with internal displacement field via gradient forms. Unlike the size-dependent differential models, the developed integral model is both self-consistent and well-posed. The governing equations and boundary conditions for the longitudinal dynamics of the rod are deduced by employing the Hamilton principle. In addition to the well-known classical boundary conditions, the developed integral rod model also contains non-classical boundary conditions. By reducing the complicated integro-differential equations to a sixth order differential equation with mixed boundary conditions, the asymptotic solutions for predicting the natural frequencies of the rods are derived for the nonlocal strain gradient rod under various boundary conditions. It is shown explicitly that the integral rod model can exert stiffness-softening and stiffness-hardening effects by considering various values of the size-dependent parameters. By studying the size-dependent effects on the longitudinal dynamics of monolayer graphene, the dispersion relation calculated by using the nonlocal strain gradient model can show good agreement with the experimental data obtained by inelastic X-ray scattering. The size-dependent effect can make monolayer graphenes possess softening frequencies.

1. Introduction

It has been recognized that the dynamic characteristics of materials and structures at micro/nano-scale are size-dependent and significantly different from their behaviors at larger scales. Therefore, an thorough understanding of the mechanical behaviors of materials and structures at micro/nano-scale is of great importance in the design and analysis of nanoelectronicmechanical (NEMS) and microelectromechanical systems (MEMS). Because molecular (atom) dynamics simulations are generally time-consuming and todays controlled experiments at micro/nano-scale are often difficult to implement, non-classical continuum mechanics have been proposed to explain and predict the scaling effects on the mechanical and physical characteristics of micro/nano-scaled materials and structures.

One of the most popular models for predicting the scaling effect is the nonlocal elasticity advanced by Eringen (2002). Unlike the classical elasticity, which assumes a point-to-point stress-strain relationship, the nonlocal elasticity is based on

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the hypothesis that nonlocal stress is influenced by the strain of all points of the body. And, such influence is captured by a spacial integral with a weighted kernel over the body, which is designed to incorporate the long-range interactions between the atoms. The nonlocal equilibrium equations are mathematically described by a set of complicated integro-differential equations. Because the source field is used as the elastic strain field, the nonlocal elasticity theory (Eringen, 2002) is often referred to as a *strain-driven nonlocal integral theory* (Romano & Barretta, 2017) or *nonlocal strain based theory* (Srinivasa & Reddy, 2017). It is suggested in Eringen (1983) that, when considering a specified kind of kernel function, the integral constitutive law of nonlocal elasticity can be cast into a differential-type constitutive equation, which is much easier to be handled than its integral counterpart. Owing to the simplicity of the differential constitutive law, the aforementioned nonlocal differential models have been extensively used to study the scaling effects on the static and dynamic behaviors of rods (Adhikari, Murmu, & McCarthy, 2014; Aydogdu, 2012), tubes (Li & Hu, 2017b), beams (Ebrahimi, Barati, & Zenkour, 2017; Lignola, Spena, Prota, & Manfredi, 2017; Mercan & Civalek, 2017; Nejad, Hadi, & Rastgoo, 2016; Reddy, 2007; Shafiei & Kazemi, 2017) and plates (Karličić, Cajić, Adhikari, Kozić, & Murmu, 2017; Li, Liu, Cheng, & Fan, 2017; Phung-Van, Lieu, Nguyen-Xuan, & Wahab, 2017). However, it has been recognized that such nonlocal differential model has some inconsistencies (Reddy & Pang, 2008) (e.g. the paradox of a cantilever beam with concentrated load). In comparison to nonlocal differential models, strain-driven nonlocal integral models have been studied by some authors and shown to be self-consistent (Fernández-Sáez, Zaera, Loya, & Reddy, 2016; Polizzotto, 2001; Romano & Barretta, 2017; Zhu, Wang, & Dai, 2017). Moreover, the confusions regarding the relations between strain-driven nonlocal differential models and integral models have been clarified recently through the analysis of beams and rods (Benvenuti & Simone, 2013; Fernández-Sáez et al., 2016; Khodabakhshi & Reddy, 2015; Wang, Zhu, & Dai, 2016; Zhu & Dai, 2012). Generally speaking, stiffness-softening effects can be found in these strain-driven nonlocal models both in differential and integral forms.

While nonlocal elastic models can only generally account for stiffness-softening effects, the gradient elasticity theory, another widely-used model, predicts a stiffness-enhancement effect (Aifantis, 1992; Mindlin, 1964). The simplified (pure) gradient elasticity theory may be developed by Aifantis (1992), in which stress is a function of strain and its gradient. The constitutive relation of the pure gradient elasticity theory is of course phenomenological, and can be referred to as a partial differential equation for strains in terms of stresses. It is showed by Srinivasa and Reddy (2017) that the strain at a point can be viewed as a functional of the entire stress field for the pure gradient elasticity theory. Therefore, the pure gradient elasticity theory may be referred to as a *stress-driven nonlocal theory*. Some works have been focused on the modified gradient elasticity models (see, e.g., Yang, Chong, Lam, & Tong, 2002). It has been reported by many works that the strain gradient effects also play a very important role in some cases when studying the scaling effect on the static and dynamic behaviors of rods (Akgöz & Civalek, 2013; Rahaeifard, 2015), beams (Akgöz & Civalek, 2011; Ghayesh, Farokhi, Hussain, Gholipour, & Arjomandi, 2016; Shenas, Ziaee, & Malekzadeh, 2016) and plates (Ghayesh et al., 2016; Guo, Chen, & Pan, 2016a). Recently, it was reported in Romano and Barretta (2017) that the strain-driven nonlocal model may admit no solution at all for the elastostatic problem of an inflected nano-beam. Some stress-driven nonlocal models proposed by Apuzzo, Barretta, Luciano, de Sciarra, and Penna (2017) and Romano and Barretta (2017) can eliminate the essential difficulty exhibited by the strain-driven nonlocal model, and provides an effective way to predict the nonlocal phenomena in small-scaled structures.

As discussed above, the scaling phenomena presented in the strain-driven and stress-driven nonlocal theories are entirely different. In this sense, a single theory that can capture both size-dependent stiffness-softening and enhancement phenomena is most desired. Recently, a mixed nonlocal theory (known as the nonlocal strain gradient theory) proposed by Lim, Zhang, and Reddy (2015) can be referred to as a simple combination of the integral constitutive equation of the strain-driven nonlocal theory and the differential constitutive equation of the stress-driven nonlocal theory. The nonlocal strain gradient model accounts for the energies diffused from surrounding particles in a reference domain by utilizing the convolution integral over nonlocal kernel functions, and can account for micro/nano-structures with internal displacement field via gradient forms. Based on the nonlocal strain gradient integral model, the closed form solution for a nonlocal strain gradient rod in tension is derived in Zhu and Li (2017a) to examine the scaling effects. When considering a specified kind of kernel function in Eringen (1983), the non integral constitutive law of nonlocal elasticity can be cast into a differential-type constitutive equation, which is much easier to be handled than its integral counterpart. With this kind of assumption, we can obtain the nonlocal strain gradient differential model, which may merely be referred to as a simple combination of the differential constitutive equations of the strain-driven and stress-driven nonlocal theories. Owing to the simplicity of the differential constitutive law, a large number of papers was written to examine the size-dependent effects on the mechanical behaviors of rods (Fernandes, El-Borgi, Mousavi, Reddy, & Mechmoum, 2017; Guo et al., 2016b; Li, Hu, & Li, 2016; Shen, Chen, & Li, 2016), beams (Barati & Zenkour, 2017; Guo, He, Liu, Lei, & Li, 2017; Li & Hu, 2016a, 2016b, 2017a; Li, Hu, & Ling, 2015; Li, Li, & Hu, 2016; Li, Li, Hu, Ding, & Deng, 2017; Lu, Guo, & Zhao, 2017; Sahmani & Aghdam, 2017a; Şimşek, 2016), shell (Mehralian, Beni, & Zeverdejani, 2017; Mohammadi, Mahinzare, Ghorbani, & Ghadiri, 2017; Sahmani & Aghdam, 2017b; Zeighampour, Beni, & Karimipour, 2017) and plates (Ebrahimi, Barati, & Dabbagh, 2016; Ebrahimi & Dabbagh, 2017a; 2017b; Karami, Shahsavari, & Janghorban, 2017; Rajabi & Hashemi, 2017; Rajabi & Hosseini-Hashemi, 2017; Xiao, Li, & Wang, 2017). As reported from these works, the nonlocal strain gradient model can predict both stiffness-softening and stiffness-enhancement phenomena depending on the specified values of the size-dependent parameters. So far, due to its simplicity, the majority of the works on nonlocal strain gradient theory are based on its differential form, and little is known about the integral form. It should be noted that nonlocal strain gradient models in differential form would reduce to nonlocal