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Exploring the entropy concept for coupled oscillators☆

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ABSTRACT

The purpose of this paper is to explore the concept of mixing entropy for systems of coupled harmonic oscillators. First, entropy is examined theoretically for a generalized oscillator configuration. A general formulation for the mixing entropy is obtained by comparing the actual entropy of a system to its hypothetical decoupled entropy. It is proven that the mixing entropy of any system is nonnegative, and that the entropy inequality property reduces to an equality with the introduction of a mixing entropy term. This result is validated using a series of numerical examples, which provide insight into the relationship between energy and entropy behavior. First, the mixing entropy is examined for an undamped system of two identical coupled oscillators. The same methodology is then applied for a system of nonidentical oscillators. Lastly, the mixing entropy is determined for a system composed of a main oscillator attached to a fuzzy structure.

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1. Introduction

Statistical energy analysis (SEA) is a method of predicting high-frequency vibroacoustic energy transmission in complex structures. Typical SEA methods apply energy balance to relate the ensemble average energy of individual structural or acoustical subsystems to the ensemble average energy flow between subsystems Lyon (1975), Fahy (1994), Burroughs (1997) and Lafont, Totaro, and Le Bot (2014). These methods are based on the first law of thermodynamics. Energy flow between coupled oscillators is an area of particular interest in the field of SEA (Mace, 2003; Mace & Ji, 2007).

Recently, the concepts of entropy and the second law of thermodynamics have been introduced for simple oscillators (Carcaterra, 1998; 2002; 2014) as well as general SEA systems (Le Bot, Carcaterra, & Mazuyer, 2010). It has been shown that entropy can provide insight into the behavior of mechanical systems. Entropy expressions have been derived for general SEA systems at steady-state, with constant power injection and dissipation effects (Le Bot, 2009; 2011). Additionally, the concept of a *mixing entropy* has been discussed in a SEA context (Le Bot, 2011), but has not been examined extensively for a more general class of vibrating systems. The use of the second law of thermodynamics completes the thermodynamic perspective of vibrating structures. This paper will expand upon the existing entropy theory for vibrating structures by investigating mixing entropy for such systems.

The first portion of this paper gives an overview of entropy. It is shown how the thermodynamic concept of entropy can be applied to vibrating mechanical systems. Following this overview, the *composite entropy* of a generalized system of oscillators is investigated. For this composite system, a *hypothetical decoupled entropy* is defined for each oscillator. The

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Nomenclature

C	arbitrary scaling constant
E	energy
E_0	initial energy
H	entropy
H^*	hypothetical decoupled entropy
M	mass of main oscillator
N	number of oscillators in the structural fuzzy system
V	equi-energy volume in phase space
$\hat{E}$	weak coupling approximation for energy
k	spring stiffness
m	oscillator mass
n	number of oscillators in the fuzzy layer
x	position degree of freedom
Ω	structure function
Φ	generating function
σ	value that minimizes the entropy
$(^*)'(x)$	derivative of $(^*)$ with respect to "x"
$(^*)_M$	main oscillator parameter
$(^*)_i$	counter for the ith layer oscillator
$(\dot{*})$	time-derivative
$:=$	equal by definition
$\equiv$	equal by enforcement

hypothetical decoupled entropy is used to show that there exists a distinguishable coupling contribution to the entropy of the system. This coupling contribution is called the *mixing entropy*, and is shown to always be nonnegative.

In the second portion of the paper, the theoretical findings of the first portion are demonstrated via several numerical examples. These examples begin with an examination of systems of identical (Fig. 1) and nonidentical (Fig. 5) coupled oscillators. Later, a system is introduced that is composed of n small oscillators that are spring-coupled in parallel to a main oscillator (Fig. 8). This system is referred to as an N-degree-of-freedom (N-DOF) fuzzy system, where $N = n + 1$ is total number of oscillators in the system. The n-dimensional subsystem of parallel oscillators is called a fuzzy structure, or layer, since it is often analyzed probabilistically (Carcaterra & Akay, 2004; Mencik & Berry, 2005; Soize & Bjaoui, 2000). Each system will be examined using a deterministic, Lagrangian mechanics approach.

2. Overview of entropy in structures

Before proceeding with the main findings of this paper, the basics of the Khinchin definition of entropy are outlined in this section for the convenience of the reader. However, it is highly recommended that readers seeking a more detailed introduction to Khinchin's entropy for vibrating structures read the work of Carcaterra (2002); 2014). Carcaterra has examined the Khinchin definition of entropy extensively for mechanical systems, and his work provides insight into the results presented in this section. Khinchin's entropy for a vibrational structure is defined as (Carcaterra, 2002; Khinchin, 1949)

$$H(E, \sigma) := \ln[e^{\sigma E}\Phi(\sigma)] + C, \tag{1}$$

where E is the system energy, C is an arbitrary scaling constant, σ is the value at which the entropy function is minimized, and $\Phi(\sigma)$ is the generating function for the system. $\Phi(\sigma)$ is related to the number of micro-states in the system and its

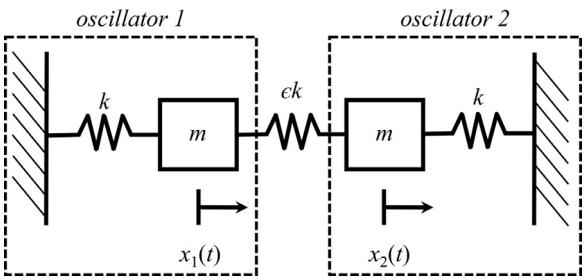

Fig. 1. System of identical coupled oscillators.

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