International Journal of Engineering Science 112 (2017) 32–41

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

A theory of dielectric fluid films between nanostructures

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ARTICLE INFO

Article history:
Received 29 January 2015
Revised 2 December 2016
Accepted 11 December 2016
Available online 20 December 2016

Keywords:
Nonlocal electromagnetic fluids
Thin dielectric fluid films
Hydrodynamic forces
Lifshitz theory
Solvation forces

ABSTRACT

In this paper a theory of dielectric fluid films between nanostructures is developed by using the theory of nonlocal electromagnetic fluids so that the nonlocal hydrodynamic force, the Lifshitz force and solvation forces can be treated systematically within the realm of continuum mechanics. In order for a continuum model to account for these surface forces of molecular origin without ad hoc assumptions, it should be endowed with a theoretical framework in which not only to represent the inner structures of fluids but to inherently possess a rational basis to merge with the Lifshitz theory without losing its rigor and also to accommodate other types of surface forces. The proposed nonlocal continuum model is formulated as such and can be adapted to all separations for which the Lifshitz theory is valid. It is applied, as an example problem, to the case of a plane-sphere configuration in the non-retarded van der Waals limit where solvation forces manifest themselves.

1. Introduction

A microscopic model for the flow behavior of a collection of molecules near a solid body was proposed by Eringen & Okada (1995) in the form of a lubrication theory for fluids with microstructure. It successfully predicted, among others, the viscosity change with the channel depth for all depths from zero to a few hundred nanometers (Israelachvili, 1986), the time evolution of the drainage of a fluid film between two molecularly smooth solid surfaces as one slowly approaches the other (Chan & Horn, 1985), and an evolution of the thickness profile for a molecularly thin lubricant on a spinning disk (Forcada & Mate, 1993). Surface forces of molecular origin and the orientation effects of molecules of an intervening fluid near solid surfaces are taken into account in the nonlocal lubrication theory of Eringen & Okada (1995) respectively through the nonlocal surface force residual at material interfaces and the response functionals pertinent to nonlocal continuum mechanics. This microscopic model however is purely mechanical in both the balance laws and constitutive equations. In order to account for physical phenomena of electromagnetic nature such as van der Waals forces, it becomes necessary to extend Maxwell's equations to nonlocal media and add to the mechanical balance laws the applied loads consisting of a sum of mechanical and electromagnetic loads. The mechanical balance laws and Maxwell's equations formulated as such constitute a complete set of balance laws for the electromagnetically active media (Eringen & Maugin, 1989, Eringen, 2002).

The most general theory of van der Waals force between any macroscopic bodies valid for any separation was developed by Lifshitz (1956) and Dzyaloshinskii, Lifshitz, & Pitaevskii (1961) and is applicable to material media with arbitrary dielectric properties at any temperature. The Lifshitz theory has attracted much attention from researchers in the fields of

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http://dx.doi.org/10.1016/j.ijengsci.2016.12.004
0020-7225/

nanomechanics, nanophotonics, biomedical sciences, etc. (Rodiguez, Capasso & Johnson, 2011) since the successful demonstration of the Casimir-Lifshitz force by Lamoreaux (1997). However, quoting Israelachvili (2011), "the only major limitation of the Lifshitz theory is that it treats the intervening solvent medium as a structureless continuum and consequently does not encompass solvation effects". At small separations, below several molecular diameters, the discrete molecular nature of a fluid medium cannot be ignored and these short-distance interactions are usually referred to as solvation forces. The solvation force is known to oscillate with distance with a spatial periodicity approximately equal to the molecular diameter, unlike the monotonic force law of the Lifshitz theory. Solvation forces are known to coexist with van der Waals forces in the short-range interactions (Israelachvili, 2011). In order for a continuum model to account for these surface forces of molecular origin in a unified manner without ad hoc assumptions, it should be endowed with a rational framework in which not only to represent the inner structures of fluids but to inherently possess a theoretical basis to merge with the Lifshitz theory without losing its rigor and to accommodate other types of surface forces as well. The theory of nonlocal electromagnetic fluids can meet both requirements, as demonstrated by Eringen & Okada (1995) and Okada (2014).

The purpose of this paper is to develop a nonlocal flow model for thin dielectric fluid films between nanostructures using the theory of nonlocal electromagnetic fluids so that the nonlocal hydrodynamic force, the generalized van der Waals force and solvation forces can be treated systematically within the realm of continuum mechanics. In Section 2 we present field equations of nonlocal electromagnetic fluids relevant to the development of our flow model. Section 3 provides a brief review of two important surface forces involved in the model, namely the generalized van der Waals force and solvation forces, showing their connections with the theory of nonlocal electromagnetic fluids. Sections 2 and 3 are provided in preparation for Section 4. In Section 4 we derive a nonlocal continuum model to predict the flow of a nonmagnetizable dielectric fluid between neutral bodies separated by a small gap to arrive at a quasi-steady equation of motion for the slowly moving surface along a line normal to both surfaces. For the purposes of illustration and verification, the proposed model is applied to the case of a plane-sphere geometry in the non-retarded van der Waals limit where the Lifshitz force and solvation force coexist.

2. Field equations

The balance laws of electromagnetically active continua are the union of laws governing electromagnetic fields and mechanical balance laws. The usual balance laws and jump conditions of nonlocal continuum mechanics therefore remain valid with the additions of electromagnetic loads and jump conditions (Eringen & Maugin, 1989, Eringen, 2002). Here we provide the relevant equations of nonlocal electromagnetic fluids adapted to the present work. It is to be remarked here that Maxwell's equations used in this paper are those formulated in Lorentz-Heaviside system of units.

Maxwell's equations with the respective jump conditions are given below. The equations of Gauss' law are

$$div\boldsymbol{D} - q_{ext} = -\hat{q} \text{ in } \mathcal{V} - \sigma, \tag{2.1}$$

$$\boldsymbol{n} \cdot [[\boldsymbol{D}]] = \hat{Q} \text{ on } \sigma \tag{2.2}$$

Here, $\mathcal{V}$ =material volume, $\boldsymbol{D}$=electric induction, q_{ext}=charge density of the external sources, [[ ]]= jump of its enclosure at a discontinuity surface σ, $\boldsymbol{n}$=positive unit normal to the surface σ, $\hat{q}$ and $\hat{Q}$ are respectively volume and surface nonlocal charge density residuals and they satisfy

$$\int_{\mathcal{V}-\sigma} \hat{q} dv = 0, \tag{2.3}$$

$$div\hat{\boldsymbol{D}} = \hat{q} \text{ in } \mathcal{V} - \sigma, \tag{2.4}$$

$$\int_{\sigma} \hat{Q} dv = 0. \tag{2.5}$$

Eqs. (2.1), (2.3) and (2.4) are valid for the entire volume $\mathcal{V}$ and its boundary $\partial\mathcal{V}$, excluding the points of the discontinuity surface σ, while Eqs. (2.2) and (2.5) hold valid on the discontinuity surface σ. $\hat{\boldsymbol{D}}$ is the nonlocal electric induction residual. It is to be remarked that while a free surface charge, a surface current and/or a surface polarization as understood in the classical sense can be prescribed on the discontinuity surface σ, we have omitted them from Eq. (2.2) in order to avoid undue complication.

The nonlocal current density residual $\hat{\boldsymbol{J}}$ is related to $\hat{\boldsymbol{D}}$ through Eq. (2.6) and subject to the restriction given by Eq. (2.7) where $\mathcal{S}$ is an open material surface with a discontinuity curve γ.

$$\hat{\boldsymbol{J}} = -\frac{\partial \hat{\boldsymbol{D}}}{\partial t}, \tag{2.6}$$

$$\int_{\mathcal{S}-\gamma} \hat{\boldsymbol{J}} \cdot d\boldsymbol{a} = 0. \tag{2.7}$$