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Indentation of a hard transversely isotropic functionally graded coating by a conical indenter

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ABSTRACT

An axisymmetric contact problem on indentation of a rigid conical punch into an elastic transversely isotropic half-space with a functionally graded transversely isotropic coating is considered. Elastic moduli of the coating vary in depth according to arbitrary continuous positive functions, independent of each other. Mathematical statement of the problem is made in terms of the linear theory of elasticity. Using integral transformation technique the problem is reduced to the solution of a dual integral equation. Kernel transform of the integral equation, which is calculated numerically from a two-point boundary value problem for a system of ordinary differential equations with variable coefficients, is approximated by a product of fractional quadratic functions. Using these approximations, an approximated solution of the problem is constructed in analytical form. The solution is asymptotically exact both for small and big values of the characteristic geometrical parameter of the problem (ratio of thickness of the coating to radius of the contact area). Approximated analytical expressions relating the displacement of the punch, indentation force acting on the punch and the size of the contact area are obtained. Correlation between the contact normal stresses arising on surface of the coated half-space and on surface of the homogeneous half-space without a coating is studied. Some relations are obtained analytically using asymptotic analysis and illustrated numerically. Results on numerical simulation of an indentation of a conical punch into a hard homogeneous or functionally graded (with linearly varying elastic moduli in depth) transversely isotropic coating are provided. The materials widely used in electronics are chosen for numerical examples. Qualitative differences in process of elastic deformation of bodies with homogeneous and functionally graded coatings are illustrated.

1. Introduction

Improvement of the service life of various structural elements is one of the most important tasks facing the modern industry. Various types of protective coatings which have a complex functionally graded or layered structure are used for this purpose. To characterize strength and elastic properties of these coatings nanoindentation experiments are widely used. Most of widely adopted nanoindentation analysis methods rely on solutions of classical contact problems for isotropic homogeneous materials. They can be used to characterize mechanic properties of homogeneous isotropic

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Nomenclature

(r', φ, z)	cylindrical coordinate system
$c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$	elastic moduli
E_{ef}	effective elastic modulus
u, w	elastic displacements in the r and z directions, respectively
$\sigma_r, \sigma_z, \tau_{rz}$	normal and tangential stress tensors components
H	thickness of the coating
a	half-width of the punch
$r = r'/a$	dimensionless radial coordinate
P	indentation force
δ	displacement of the punch downward the z direction
λ	ratio of the coating thickness to the half-width of the punch

coatings under certain experiment conditions, but have little to do with depth-wise inhomogeneous or anisotropic coatings. Increasing usage of complex functionally graded and composite anisotropic coatings requires development of more accurate mathematical models for analysis of experimental results.

Indenters of different sizes and shapes are used for indentation of samples. The most common shapes of the indenter's tip are spherical, pyramidal and conical. It is known that in mathematical simulation of the widely used Berkovich pyramid a conical punch with an appropriate opening angle can be successfully used. This paper is focused on mathematical simulation of indentation of a conical punch into an elastic transversely isotropic half-space with a functionally graded transversely isotropic coating, when each elastic modulus of the coating vary with depth according to arbitrary continuous positive functions independent of each other.

Closed-form solution for contact problem on conical punch indentation into a homogeneous half-space was obtained for a case of isotropic material (Sneddon, 1951; Staerman, 1949) as well as for transversely-isotropic piezoelectric materials (Chen, Shioya, & Ding, 1999; Giannakopoulos & Suresh, 1999). Delafargue and Ulm (2004) derived explicit solutions for the indentation moduli of a transversely isotropic homogeneous medium and a general orthotropic homogeneous medium under rigid conical indentation. Displacements, stresses and strains inside the half-space were also obtained in closed analytical form for normal indentation of a conical (and also for flat or spherical) punch without friction by Karapetian, Kachanov, and Kalinin (2005) and for arbitrary planform punches by Karapetian, Kachanov, and Kalinin (2009). Effect of frictional sliding was considered by Makagon, Kachanov, Karapetian, and Kalinin (2007).

Oliver and Pharr (1992) proposed and improved (2004) method for determination of hardness and Young's modulus of a homogeneous isotropic material, based on Sneddon's solution for an axisymmetric indenter. It has been shown theoretically and experimentally that such method can be used to characterize properties of homogeneous coatings under certain experiment conditions. When applied to functionally graded or nanocomposite coatings, sometimes it can be used to give integral mechanical characteristics, suitable for prediction of their operational performance (Varavka, Kudryakov, Ryzhenkov, Kachalin & Zilova, 2014; Kudryakov & Varavka, 2015). However, accurate characterization of thin films or inhomogeneous coatings requires methods relying on more advanced mathematical models. Goryacheva, Torskaya, Kornev, Kovaleva, and Myshkin (2015) used the solution for a two-layered elastic half-space to characterize properties of bicomponent metal vapor deposited coatings.

Contact problems for a half-space with functionally graded coatings or multilayered coatings with constant elastic moduli in each layer present a difficult mathematical challenge. Usually such a problem is reduced to the integral equations requiring specific methods to solve. For thick coatings regular asymptotic method was successfully used (Alexandrov, 1998; Vorovich & Ustinov, 1959). The solution of the problem on indentation of a homogeneous layer by a punch was presented in terms of series with respect to the ratio of the punch radius to the layer thickness (referred to as parameter λ). The solution based on the Wiener–Hopf factorization effective for small thickness of a coating was constructed in (Alexandrov, 1998; Alexandrov & Vorovich, 1964).

Contact problems for transversely isotropic functionally graded coatings are actively being investigated nowadays. Indentation of punches with different geometries into an elastic substrate with a homogeneous coating was studied by Argatov and Sabina (2012, 2013, 2016), in particular, the asymptotical modeling for frictionless or adhesive no-slip contact for thin or thick coating was provided. Plane and axisymmetric contact of a rigid conductive punch with a transversely isotropic piezoelectric half-plane and a half-space with a functionally graded coating with exponentially varying elastic moduli was considered by Ma, Ke, and Wang (2014) and Su, Ke, and Wang (2016). Plane thermoelastic frictional contact of functionally graded materials with arbitrarily varying thermoelastic properties was studied by Liu, Ke, Wang, Yang, and Alam (2012). Integral equation of the problem was solved using collocation method. The solution of the integral equation is approximated by a function containing a finite number of parameters. By satisfying the integral equation at a finite number of points a system of linear algebraic equations is obtained to determine these parameters. Plane contact of a rigid punch with a functionally graded coating/substrate system assuming exponential variation of elastic moduli in the coating with and without frictional heating was also studied by Alinia, Beheshti, Guler, El-Borgi, and Polycarpouc (2016) and Guler and Erdogan (2004).