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Size-dependent vibration analysis of nanobeams based on the nonlocal strain gradient theory

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ABSTRACT

A size-dependent sinusoidal shear deformation beam model is developed to investigate the free vibration of nanobeams based on the nonlocal strain gradient theory. The new model contains a nonlocal parameter and a material length scale parameter which can capture the size effect. The governing equations and boundary conditions are derived by employing Hamilton's principle. Navier's method is utilized to obtain analytical solutions for natural frequencies of simply supported nanobeams. The results are compared with other beam models and other classical and non-classical theories. Several numerical examples are presented to illustrate the effects of nonlocal parameter, material length scale parameter, slenderness ratio and shear deformation on the free vibration of nanobeams. It is found that natural frequencies predicted by the nonlocal strain gradient theory are higher than those predicted by nonlocal theory and lower than those obtained by strain gradient theory. When the length scale parameter is smaller than the nonlocal parameter, the nanobeam exerts a stiffness-softening effect. When the length scale parameter is larger than the nonlocal parameter, the nanobeam exerts a stiffness-hardening effect. Moreover, it is observed that the effect of shear deformation becomes more significant for nanobeams with lower values of slenderness ratios and for higher modes.

1. Introduction

Nanostructures such as nanorods, nanobeams and nanoplates have been widely used in nanoelectromechanical systems (NEMS) (Craighead, 2000; Ekinci & Roukes, 2005) due to their extraordinary mechanical, electrical and thermal properties. However, significant size dependent effects on the mechanical and physical properties of small-scaled structures have been observed in many experimental investigations and atomistic simulations. Therefore, size effect must be taken into account for better understanding the mechanical behaviors of these small-scaled structures. It is known that classical continuum theory is size-independent and cannot capture the size effect. To overcome this problem, several non-classical continuum theories involving additional material length scale parameters were developed, such as nonlocal elasticity theory (Eringen,1972,1983), strain gradient theory (Mindlin, 1964,1965) and nonlocal strain gradient theory (Lim, Zhang, & Reddy, 2015).

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According to the nonlocal elasticity theory proposed by Eringen (1972, 1983), the stress at a reference point in an elastic continuum not only depends on the strain at the point but also on strains at every point of the body. In the context of nonlocal elasticity theory, numerous studies have been performed to study the static and dynamic behaviors of nanobeams (Ansari, Oskouie, Gholami, & Sadeghi, 2016; Aydogdu, 2009a; Ebrahimi & Salari, 2015a, b; Eltaher, Mahmoud, Assie, & Meletis, 2013; Nejada, Hadi, & Rastgoo, 2016; Peddieson, Buchanan, & McNitt, 2003; Rahmani & Pedram, 2014; Reddy, 2007; Simsek & Yurtcu, 2013; Thai, 2012; Thai & Vo, 2012; Wang, 2005). Although the nonlocal elasticity theory is widely used to capture the size effect, it characterizes only stiffness softening effect. The stiffness enhancement effect reported in many experimental and theoretical works (Fleck & Hutchinson, 1993; Lam, Yang, Chong, Wang, & Tong, 2003; Stolken & Evans, 1998) cannot be incorporated by Erigen's nonlocal theory.

The strain gradient theory (Mindlin, 1964, 1965) is a microstructure-dependent continuum theory which can capture the stiffness enhancement effect. It states that the total stress field must account for additional strain gradient terms to consider microstructural deformation mechanism. On the basis of Mindlin's strain gradient theory, Lam et al. (2003) developed the modified strain gradient theory which contains only three additional material length scale parameter besides two classical ones. Accordingly, Yang, Chong, Lam, and Tong (2002) elaborated the modified couple stress theory which contains only one additional material length scale parameter. In fact, the modified couple stress theory can be considered as the special case of the modified strain gradient theory. Within the framework of strain gradient theory and modified couple stress theory, there have been a considerable amount of researches on the static and dynamic behaviors of small-scaled structures (Akgoz & Civalek, 2011, 2014a; Al-Basyouni, Tounsi, & Mahmoud, 2015; Ansari, Gholami, & Sahmani, 2011; Attia & Mahmoud, 2016; Ghayesh, Amabili, & Farokhi, 2013; Kong, Zhou, Nie, & Wang, 2009; Li, Feng, & Cai, 2014; Ma, Gao, & Reddy, 2008; Mohammadimehr, Farhi, & Alimirzaei, 2016; Nateghi, Salamat-talab, Rezapour, & Daneshian, 2012; Park & Gao, 2006; Roque, Fidalgo, Ferreira, & Reddy, 2013; Wang, Zhao, & Zhou, 2010). These works showed that microstructure effect plays a significant role in modeling the small-scaled structures.

From the mentioned above, it is clear that the nonlocal elasticity theory and the strain gradient theories describe two entirely different physical characteristics of materials and structures at small scale. The nonlocal elasticity theory does not include nonlocality of higher-order stresses while the strain gradient theory only considers local higher-order strain gradients without nonlocal effects. To assess the true effect of the two length scales on the mechanical and physical responses of size-dependent small-scaled structures, Lim et al. (2015) combined the classical nonlocal elasticity with strain gradient theory and resulted in a higher-order nonlocal strain gradient theory, which reasonably explains the size-dependent wave propagation behavior of carbon nanotubes and results in an excellent matching with those of molecular dynamics simulations. Based on the nonlocal strain gradient theory, several studies have been performed to investigate the mechanical behavior of small-scaled structures (Ebrahimi & Barati, 2017a, b; Ebrahimi, Barati, & Dabbagh, 2016; Farajpour, Yazdi, Rastgoo, & Mohammadi, 2016; Li & Hu, 2015, 2016; Li, Hu, & Li, 2016; Li, Li, & Hu, 2016; Simsek, 2016;). For example, Li et al. (2016) developed a size-dependent Timoshenko beam model in the framework of nonlocal strain gradient theory to investigate the free vibration of functionally graded nano/micro-scaled beams. Simsek (2016) studied the nonlinear free vibration of functionally graded nanobeam using the nonlocal strain gradient theory and Euler-Bernoulli beam model. Ebrahimi et al. (2016) examined the wave propagation behavior of an inhomogeneous functionally graded nanoplate subjected to nonlinear thermal loading by the means of nonlocal strain gradient theory.

On the other hand, various beam theories have been proposed to study the mechanical behaviors of beams. Among them, Euler-Bernoulli beam theory (EBT) and Timoshenko beam theory (TBT) are more widely used. EBT neglects the shear deformation, which can be suitable for slender beams. However, effects of shear deformation become more prominent for moderately thick beams. TBT takes account for effects of shear deformation and assumes that transverse shear stress and strain are invariant along the thickness of the beam. Actually, there are no transverse shear stress and strain at the top and bottom surfaces of the beam and their distributions are not uniform. For this reason, a shear correction factor is required in TBT. After that, higher-order shear deformation beam theories including parabolic (third-order) beam theory (Reddy, 1984), trigonometric (sinusoidal) beam theory (SBT) (Touratier, 1991), hyperbolic beam theory (Soldatos, 1992), exponential beam theory (Karama, Afaq, & Mistou, 2003) and a general exponential beam theory (Aydogdu, 2009b) have been proposed to satisfy the zero transverse shear stress and strain condition at the top and bottom surfaces of the beam without any shear correction factors. On the basis of these higher-order theories, several size-dependent beam models have been developed based on the strain gradient theory and modified couple stress theory (Akgoz & Civalek, 2013, 2014b, c, d; Lei, He, Zhang, Gan, & Zeng, 2013; Simsek & Reddy, 2013; Mohammad-Abadi & Daneshmehr, 2014). Recently, Akgoz and Civalek (2015) developed a new size-dependent beam model on the basis of hyperbolic shear deformation beam and modified strain gradient theory. Kolahchi and Bidgoli (2016) developed a size-dependent sinusoidal beam model to study the dynamic instability of single-walled carbon nanotubes based on the modified couple stress theory. However, size-dependent higher-order beam models in conjunction with the newly proposed nonlocal strain gradient theory have not been studied.

In present work, based on the nonlocal strain gradient theory, a size-dependent sinusoidal shear deformation beam model is developed for the free vibration analysis of nanobeams. The new model contains a nonlocal parameter and a material length scale parameter which can capture the size effect. The governing equations and boundary conditions are derived by employing Hamilton's principle. Navier's method is utilized to obtain analytical solutions for natural frequencies. The results are compared with other beam theories as EBT and TBT and other classical and non-classical theories as classical theory, nonlocal elasticity theory and strain gradient theory. The influences of nonlocal parameter, material length scale parameter, slenderness ratio and shear deformation on the vibration of nanobeams are examined in detail.