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The response of conductive-fiber reinforced composites to electric field

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ABSTRACT

An analytical procedure, which couples electric, magnetic, thermal and mechanical effects is presented for the prediction of the response of unidirectional fiber-reinforced composites that are subjected to electric field, applied in the fibers' direction. It is assumed that at least one of the phases of the composite (e.g., the fibers) is electrically conductive, and that all phases are thermally conductive. The composite is assumed to occupy a finite, symmetric domain which is discretized into a double array of subcells. The governing equations, with the interfacial and boundary conditions, are satisfied in the integral sense. The externally applied field generates electric current, which induces a magnetic field as well as temperature increase. The mechanical deformation of the composite results from the combined effect of the ponderomotive force, which is created by the magnetic field, and the temperature distribution within the constituents. The purpose of the present paper is three-fold. (a) To perform quantitative analysis of the model for the ponderoromotive force in deformable media. (b) To present computational strong-form treatment of the magneto-mechanical boundary-value problem in a composite. (c) To suggest a computational apparatus for deriving the response of a sensor/actuator excited by an applied electric field or electric field gradient. Application is given, presenting the magnetic, thermal and mechanical field distributions, as well as the macroscopic (global) response of a composite, which consists, for simplicity, of two iron fibers embedded in an epoxy matrix.

1. Introduction

The micromechanical response of composite materials is usually investigated when the application of mechanical or thermo-mechanical loading is considered. Additional macroscopic physical effects, such as electric and magnetic field distribution, are commonly treated for materials with constitutive coupling, such as in the case of piezoelectric and electrostrictive or magnetostrictive materials. Fewer work deals with the response to electric loading of electrically conductive composites, as, for example, in the case of a composite in which one of the phases is electrically conductive (e.g., carbon fibers). In such a case, as a result of the application of electric field, electric current flows in the conductive phase, inducing magnetic field distribution, as well as temperature increase. The magnetic field gives rise to the so-called ponderomotive, or generalized Lorentz force, which, along with the corresponding temperature distribution, generates mechanical deformation.

The main challenge in addressing the aforementioned type of problems is the proper description of Maxwell's equations and the associated Maxwell tensor, or alternatively, the ponderomotive force and moment couple density, for the case of

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deforming continua. Fundamental investigations can be found in the work of Tiersten (1971), Pao (1978) and in the recent reviews of Hutter, van de Ven, and Ursescu (2006) and Santapuri, Lowe, Bechtel, and Dapino (2013).

Beyond the establishing of the constitutive and governing equations of the aforementioned 'multi-physics' problem, there arise mathematical issues pertinent to the specifics of the field form of Maxwell's equations. In the case of slow electromagnetic dynamics, the main issue is related to the fact that the electric and magnetic fields each have to satisfy multiple differential equations. In the context of numerical solution, this difficulty is usually treated by introducing auxiliary gauges, e.g. the Coulomb gauge, see Zienkiewicz, Lyness, and Owen (1977), Simkin and Trowbridge (1979) and Barton and Cendes (1987), for example. Another approach is taken in Kudryavtsev and Trashkeev (2013), where a hyperbolic system of evolution equations is derived with no auxiliary differential equations to impose.

Computationally, the coupled multiphysics response of composite materials for general three-dimensional domains is normally derived using the finite element formalism, see (Sridhar, Keip, & Miehe, 2015) for a recent example. In the case of plates, the coupled problem is solved by integrating the plate equations using finite difference discretization, in conjunction with the Love-Kirchhoff, as in Zhupanska and Sierakowski (2007a), Zhupanska and Sierakowski (2007b), Barakati and Zhupanska (2014) or the von Kármán approximation, as in Librescu, Hasanyan, Qin, and Ambur (2003).

In Librescu et al. (2003), the dynamic response of a plate reinforced by electrically conducting fibers is considered. As a result of the nonlinear dependence of the mechanics on the magnetic field, linearization is performed, based on the assumption of small disturbances. In Barakati and Zhupanska (2014) on the other hand, a similar setting is assumed, also for plate dynamics, however without the small disturbances assumption. A different model for the ponderomotive force is used, and much as in Librescu et al. (2003), uniform material, rather than a composite, is considered. The solution is obtained by a finite difference method.

The purpose of the present article is three-fold. First, to perform quantitative investigation of the effect of the correction for the ponderomotive force, relatively to the standard expression of the Lorentz force, which is established for the particle-in-vacuum case. To this end we follow (Santapuri et al., 2013) in choosing the Maxwell–Minkowski model, reduced for the linear quasistatic regime. The framework for the examination of the aforementioned effect is that of unidirectional deformation of a two-constituent composite specimen. The second perspective in which the present work is performed is computational. To this end, the Higher Order Theory (HOT), as in Aboudi, Arnold, and Bednarcyk (2013), which has been employed for the analysis of functionally graded materials, is generalized to account for electric-current-induced mechanical deformations. In the framework of the HOT, the governing equations together with the interfacial and boundary conditions are imposed in the average (integral) sense. It should be noted that since the HOT is a strong-form approach to the solution of boundary-value problems, the present work opts to construct a strong-form computational apparatus for the analysis of the multi-physics response of composites in the linear quasistatic regime. In the third perspective, the present paper in effect proposes a sensor/actuator with the accompanying computational apparatus, which will have the effect of nonlinear response to either electric field or electric field gradient, in line with the view presented in Santapuri, Lowe, and Bechtel (2015). The details of these perspectives are further discussed in the concluding section.

To elaborate on the third perspective, the present article offers a coupled electro-magneto-thermo-mechanical analysis for the prediction of the mechanical deformation of partially conductive composites, caused by the application of electric field. The engineering motivation for this analysis can be illustrated by the following discussion. Consider a setting (e.g. a large capacitor), in which a uniform electric field exists. It is then assumed that a local effect (perturbation) (a small relative change in the field but with a high local gradient, as created by a small conducting object) has been introduced, as a result of which the electric field loses its uniformity. The present analysis shows that a device (sensor) in the form of a ring that consists of, say, two conductive wires embedded in a polymeric (insulation) matrix, will reveal the effect by showing the induced mechanical deformation. This deformation, caused jointly by the Joule heating and the ponderomotive force, can be calibrated to estimate the intensity of the perturbation-induced local electric field gradient. Such a (partially conducting) ring would be advantageous over, say, a piezoelectric sensor in that it would sense the finite electric field gradient rather than the nearly uniform local field itself. High accuracy of the device-calibration would be obtained due to the offered analysis, owing to it being based on strong-form solution of the equations (which is possible for linear constitutive relations). It should be noted that a sensor in the form of a ring is a three-dimensional object, and a 3D imperfectly conducting ring in a non-uniform electric field develops a current, see (Assis, Rodrigues, & Mania, 1999). It is also shown in that article that the expressions for the 3D corrections to the electric field in the ring become negligible for a slender wire. Consequently, in the limiting case of a slender ring-wire the applicability of the present 2D analysis is justified.

In the present article, the analysis of mechanical deformation generated by the application of externally applied electric field on electrically and thermally conducting fiber-reinforced composites is considered. The composite material is assumed to consist of continuous fibers that are distributed in the matrix, forming a symmetric array. The electric field is applied in the fibers' direction, and the boundaries of the composite are assumed to be traction-free and thermally convective. The composite region is divided into a double array of subcells. The magnetic field induced by the applied electric field is derived, and the resulting ponderomotive force is established. Next, the equations that govern the thermal problem related to the heat generated by the electric current are established and solved numerically, to obtain the temperature field distribution and its gradient. Finally, the mechanical deformations of the composite phases are obtained from the solution of a system of sparse algebraic equations, following the HOT approach. The presented derivation is illustrated for the case in which uniform (throughout the composite domain) electric field is applied to a thermally conductive polymeric matrix (epoxy) reinforced by two iron fibers, which are both electrically and thermally conductive. The solution demonstrates the