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Three-dimensional exact magneto-electro-elastic field in an infinite transversely isotropic space with an elliptical crack under uniform loads: Shear mode

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ABSTRACT

This paper studies an elliptical planar crack embedded in an infinite transversely isotropic medium in the framework of magneto-electro-elasticity. The crack is assumed to be subjected to uniformly distributed shear loads, which are anti-symmetrical with respect to the crack plane. The boundary integral equation is established via the general solution in conjunction with the generalized method of potential theory. Exact and complete field variables are obtained in terms of elliptical functions. Important parameters in fracture mechanics, e.g., the crack slip displacement, the shear stress at the crack front, and the corresponding stress intensity factor, are explicitly derived. The corresponding solutions, in the context of piezoelectricity, piezomagnetism and pure elasticity are also presented, as a byproduct of the present work. The present closed-form analytical solutions may serve as benchmarks for future simplified and numerical studies.

1. Introduction

The pioneering work on the magneto-electro-elastic materials (or multiferroic composite media) may retrospect to the 70s of the last century (Van Suchtelen, 1972). The magneto-electro-elastic composite materials, which simultaneously combine piezoelectric and piezomagnetic phases, exhibit interaction among the magnetic, electric and elastic fields (Davis, 1974; Van Den Boomgaard, Vanrun, & Van Suchtelen, 1976). At room temperature, the coupling effect is much stronger than that of a single phase piezoelectric material or piezomagnetic material. This leads to a high sensitivity and remarkable conversion efficiency for three fields (Bichurin, Petrov, Priya, & Bhalla, 2012; Vanrun, Terrell, & Scholing, 1974). With the properties of lightweight and high strength, multiferroic composite materials are extensively used in a variety of engineering fields, such as space plane, supersonic aircraft, rockets, spacecraft, nuclear reactors, nuclear submarines, electronic packaging, and

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so forth (Chen, Pan, Wang, & Zhang, 2010; Eerenstein, Mathur, & Scott, 2006; Li, Wu, Jin, & Chen, 2015; Nan, Bichurin, Dong, Viehland, & Srinivasan, 2008; Zhai, Dong, Xing, Li, & Viehland, 2007; Zheng et al., 2004).

The multiferroic composite materials are brittle at room temperature and susceptible to fracture when subjected to electrical, magnetic and mechanical loads in service (Chen, 2015; Feng & Pan, 2008; Gao, Kessler, & Balke, 2003; Zhou, Zhang, & Wu, 2007). Consequently, fracture analyses are of great importance for structural design and crack detection. Crack analyses play an important role in health monitoring of advanced equipments in high-tech fields. In the magnetic field probes and ultrasound equipments, latent cracks may cause electric, magnetic and elastic energy aggregation, giving rise to unreliable results (Zhou et al., 2007). For the sake of the requirement of high strength, high reliability and longer service life, it is necessary to understand the crack behavior of multiferroic composite materials.

On account of the engineering significance and academic value, the crack problems of magneto-electro-elastic composite materials have attracted considerable interest. To date, intensive scientific efforts have been made on the 2D crack problems during the past three decades, by generalizing various classical treatments in planar elasticity; for instance, making use of the Laplace transform, the Fourier transform and the dislocation density functions, Feng and his coauthors conducted a series of studies on dynamic and transient responses of structures with interfacial crack (Feng & Pan, 2008; Feng & Su, 2006, 2007; Feng, Xue, & Zou, 2005; Li, Xu, & Feng, 2011; Su, Feng, & Liu, 2007), and obtained the field intensity factors and energy release rates. Gao, Tong, and Zhang (2004), developing a Stroh-type formalism for anti-plane deformation of a cracked structure, investigated a defect problem, where an elliptical cavity is assumed to be embedded in an infinite medium subjected to remote uniform in-plane electromagnetic and/or anti-plane mechanical loads. The explicit expressions for the mode III stress intensity factors of a crack, as a limiting geometry of the elliptical cavity, were obtained in closed-form.

Employing the complex variable approach in conjunction with the conformal mapping technique, Soh and Liu (2004) studied the mode III problem of an interfacial edge crack in a magneto-electro-elastic bimaterial. Making use of the Fourier transform technique, Zhou, Wu, and Wang (2005) obtained a closed-form solution of a crack in magneto-electro-elastic composites under anti-plane shear stress loads on the permeable crack surface. The problem for a defect in an infinite magneto-electro-elastic solid induced by the thermal analog of a line temperature discontinuity and a line heat source was researched by Qin (2005), and the closed-form Green's functions were obtained by virtue of the Stroh formalism, the conformal mapping, and the perturbation technique. Employing the integral transform and dislocation density functions, Ma, Li, Abdelmoula, and Wu (2007) studied the mode III problem of a crack embedded in the functionally graded strip of magneto-electro-elastic material. It should be pointed out that a thorough literature survey is far beyond the scope of the present work and the interested readers may refer to the papers cited in the above-mentioned references.

As a matter of fact, crack problems should be of three-dimensional nature in practice. In comparison to planar or anti-plane analyses, 3D crack problems are more complicated in mathematic and physics, leading to few academic effort by the end of 20th century. As a reduced 3D situation, axisymmetric crack problems began to draw academic attention ten year ago; for instance, introducing two displacement functions and utilizing rigorous operator theory and the generalized Almansi theorem, Chen, Lee, and Ding (2004) developed the static general solution expressed by six harmonic functions. A penny-shaped crack problem is solved by means of the general solution and the generalized method of potential theory. Magneto-electro-elastic field variables and some important parameters for mode I crack problem in fracture mechanics were presented in terms of elementary functions. Following the method in Kogan, Hui, and Molkov (1996), Zhao, Yang, and Liu (2006) obtained the exact solution for a penny-shaped crack contained in an infinite transversely isotropic magneto-electro-elastic medium as the degenerated situation of an ellipsoidal cavity under remotely exerted axisymmetric loads. Niraula and Wang (2006a; 2006b) studied a penny-shaped crack subjected to temperature variations and uniform heat flows, respectively, and obtained the generalized stress intensity factors, by virtue of the Hankel integral transform. Rogowski (2014) analyzed a mode I penny-shaped crack with conducting magneto-electric boundary conditions embedded in a magneto-electro-elastic layer and obtained field intensity factors, by means of the Hankle transform technique as well.

The problem of elliptical crack under normal and shear loads is of scientific and engineering significance. A great deal of academic effort has been made in the past (Green & Sneddon, 1950; Kassir & Sih, 1966; Shah & Kobayashi, 1971; Vijayakumar & Atlurl, 1981; Willis, 1968; Zhao, Wu, & Yan, 1989; Zhu, Li, & Chao, 2001). Most of the pioneering works were devoted to physical parameters on the crack plane, e.g., the stress distributions at the crack front, crack opening/slip displacement and stress intensity factor in the context of pure elasticity. For an elliptical crack embedded in an infinite isotropic elastic medium, Kassir and Sih (1975) presented the field variables in response to the various types of external stimulus in terms of elliptical functions. However, the complete elastic field of a cracked anisotropic body, had not been presented by the end of the last century. This situation was changed by Fabrikant (2004), who employed the method of potential theory (Fabrikant, 1989, 1991) and obtained the 3D elastic field in an infinite transversely isotropic space containing an elliptical crack subjected to uniform tension and shear force. Nevertheless, the stress intensity factor at the crack tip is not discussed. Generalizing the potential theory to magneto-electro-elasticity, Li, Zheng, Kang, Chen, and Müller (2016) systematically studied the mode I problem of an elliptical or penny-shaped crack and presented the exact and closed-form 3D solutions.

According to the literature survey, an elliptical crack contained in an infinite transversely isotropic space of multiferroic composite medium, which is subjected to uniform shear loads anti-symmetrically exerted on the crack surfaces has not been studied yet. The present work may serves as a natural continuation of the study of Li et al. (2016).

The structure of this paper is as follows. In Section 2, the basic equations and static general solution of magneto-electro-elastic material with transverse isotropy are presented. Section 3 is devoted to formulating the problem with the