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Closed form solution for a nonlocal strain gradient rod in tension

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ABSTRACT

A size-dependent integral elasticity model is developed for a small-scaled rod in tension based on the nonlocal strain gradient theory. The integral rod model contains a nonlocal parameter and a material length scale parameter to incorporate the scaling effects of nonlocal stress and microstructure-dependent strain gradient. In comparison to size-dependent differential models, the developed integral rod model is both self-consistent and well-posed. The governing equations and boundary conditions for the nonlocal strain gradient rod in tension are derived by employing the principle of virtual work. In addition to the classical natural and essential boundary conditions, non-classical natural and essential boundary conditions are present for the integral rod model. The closed-form solutions for predicting the displacement and reduced Young's modulus are derived for four types of boundary conditions. It is shown explicitly that the integral rod model can exert stiffness-softening and stiffness-hardening effects by considering various values of size-dependent parameters and boundary conditions. It is found that, the developed rods with four different boundary conditions can predict the scaling effects of the Young's modulus of single-walled carbon nanotube, and the scaling effects are more sensitive to the size-dependent parameters (the material length scale parameter and the nonlocal parameter) in comparison with the non-classical boundary conditions.

1. Introduction

Classical elasticity theories describe a structure in terms of some variables (including mass, stiffness, temperature, strain and stress), which are suited directly to measurements and senses. The expediency, practicality and successes of classical elasticity mechanics have being made elasticity models useful to explain and predict physical phenomena throughout the history of engineering sciences. The impetus for contemporary micro/nano-scaled non-classical mechanics is primarily owing to the todays nano-technologies, nano-engineering, nano-science and bio-medical technology (e.g. computer-aided material design, sub-micro scaled sensors, motors, the mechanics of nano-tubes/wires, thin film, manufacturing devices and structural components of nanoelectromechanical and microelectromechanical systems, and micro-biophysics/biochemistry systems). Due to the fact that the molecular (atom) dynamics simulations are time-consuming and todays controlled micro/nano-

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scaled experiments are usually difficult to implement, some non-classical continuum mechanics have been developed for explaining and predicting the size-dependent mechanical and physical characteristics of micro/nano-scaled structures.

In the framework of nonlocal elasticity advanced by Eringen (2002), each particle inside a material (or structure) can receive energies diffused from all other particles in a reference domain such that the long-range forces (or the non-locality effects) of obstacles (e.g., holes or cracks) can be circumvented. According to the non-locality hypothesis, the nonlocal equilibrium equations have a similar formula with classical elasticity models, but the total fundamental field can account for both local and residual-nonlocal fields. When considering a specified kind of nonlocal kernel functions (Eringen, 1983), the integral constitutive law of nonlocal elasticity can be cast into a differential-type constitutive equation, which is much easier to be used for analyzing the static and dynamic behaviors than its integral counterpart. Owing to the simplicity of the differential constitutive law, the aforementioned nonlocal differential models have been extensively used to study the scaling effects on the static and dynamic behaviors of rods (Adhikari, Murmu, & McCarthy, 2014; Aydogdu, 2012), beams (Ebrahimi, Barati, & Zenkour, 2017; Lignola, Spena, Prota, & Manfredi, 2017; Mercan & Civalek, 2017; Nejad, Hadi, & Rastgoo, 2016; Reddy, 2007; Shafiei & Kazemi, 2017) and plates (Karličić, Cajić, Adhikari, Kozić, & Murmu, 2017; Li, Liu, Cheng, & Fan, 2017; Phung-Van, Lieu, Nguyen-Xuan, & Wahab, 2017). However, it has been recognized that such nonlocal differential model has some inconsistencies (Reddy & Pang, 2008) (e.g. the paradox of a cantilever beam with concentrated load). In comparison to nonlocal differential models, nonlocal integral models have been studied by some authors and shown to be self-consistent (Altan, 1989; Eringen, 2002; Fernández-Sáez, Zaera, Loya, & Reddy, 2016; Pisano & Fuschi, 2003; Polizzotto, 2001; Romano & Barretta, 2017). Moreover, the confusions regarding the relations between nonlocal differential models and integral models have been clarified recently through the analysis of beam structures (Fernández-Sáez, Zaera, Loya, & Reddy, 2016; Khodabakhshi & Reddy, 2015; Wang, Zhu, & Dai, 2016). We also note that the mechanical behaviors of a small-scaled rod in tension modeled using nonlocal differential models and integral models have been found to be quite different (Benvenuti & Simone, 2013; Zhu & Dai, 2012). Generally speaking, stiffness-softening effects can be found in these nonlocal elasticity models both in differential and integral forms.

The gradient elasticity theory (Mindlin, 1964) hypothesized that a material at nano/micro-scale should be not just modeled as collections of points and has to be considered as micro-continua with internal displacement field via gradient forms. Some works have been focused on the simplified and modified gradient elasticity models (see, e.g., Aifantis, 1992; Yang, Chong, Lam, & Tong, 2002) It has been reported by many works that the strain gradient effects also play a very important role in some cases when studying the scaling effect on the static and dynamic behaviors of rods (Akgöz & Civalek, 2013; Rahaeifard, 2015), beams (Akgöz & Civalek, 2011; Ghayesh, Farokhi, & Alici, 2016; Shenas, Ziaee, & Malekzadeh, 2016) and plates (Ghayesh, Farokhi, & Alici, 2016; Guo, Chen, & Pan, 2016), and the stiffness-enhancement effects have been often reported in these strain gradient models.

As discussed above, the mechanism analysis presented in nonlocal theory and strain gradient theory are entirely different: the nonlocal theory concludes that “smaller is more compliant”, while the strain gradient theory says the opposite: “smaller is stiffer”. However, it has been reported by some experimental observations that stiffness-softening and enhancement phenomena may be both observed, depending on the feature of microstructures at nano-scale (Li, Wei, Lu, Lu, & Gao, 2010; Tian et al., 2013). In this sense, a single theory that can capture both size-dependent stiffness-softening and enhancement phenomena is most desired. Recently, Lim, Zhang, and Reddy (2015) presented a nonlocal strain gradient theory which hypothesizes that the stress tensor should account for the effects of both nonlocal stress tensor and strain gradient stress tensor, and hence bring nonlocal theory and strain gradient theory into a single theory. Based on this model, many authors have examined the size-dependent effects on the mechanical behaviors of rods (Fernandes, El-Borgi, Mousavi, Reddy, & Mechmoum, 2017; Guo et al., 2016; Li, Hu, & Li, 2016; Shen, Chen, & Li, 2016), beams (Barati and Zenkour, 2017; Li and Hu, 2016a; 2016b; 2017; Li, Hu, & Ling, 2015; Li, Li, Hu, Ding, & Deng, 2017; Şimşek, 2016) and plates (Ebrahimi, Barati, & Dabbagh, 2016; Ebrahimi & Dabbagh, 2017a; 2017b). As reported from these works, the nonlocal strain gradient model can predict both stiffness-softening and stiffness-enhancement phenomena depending on the specified values of the size-dependent parameters. So far, due to its simplicity, all the works on nonlocal strain gradient theory are based on its differential form, and little is known about the integral form. Also, it should be noted that nonlocal strain gradient models in differential form would reduce to nonlocal differential models when neglecting the strain gradient stress tensor, which implies that such differential models may also not be self-consistent.

In this paper, we confine our attention to a nanorod in tension. A size-dependent integral elasticity model will be developed for small-scaled rods in tension based on the nonlocal strain gradient theory. The governing equations and boundary conditions for the nonlocal strain gradient rod in tension will be derived by employing the principle of virtual work. The closed-form solutions for predicting the displacement and reduced Young’s modulus will be derived for four types of boundary conditions. It will be shown that the integral rod model can exert stiffness-softening and stiffness-hardening effects by considering various values of size-dependent parameters and boundary conditions.

2. Governing equation of a nonlocal strain gradient rod in tension

A uniform rod with length L and cross-sectional area A is considered in this study, as plotted in Fig. 1. The case of the stretching with a constant static force P of small-scaled straight rod will be studied. The stretch occurs along the x direction, and the deformation of the cross section in the radius direction is assumed to be negligible. In this section, the governing equation for a small-scaled straight rod in tension will be derived in the framework of the nonlocal strain gradient theory.