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Plane strain dynamic responses of a multi-layered transversely isotropic saturated half-space

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ABSTRACT

An exact dynamic stiffness matrix method is proposed to evaluate the plane strain responses due to time-harmonic loads and pore fluid pressure applied in the interior or on the surface of a multi-layered transversely isotropic (TI) saturated half-space. First, the governing equations of the TI saturated medium are solved analytically by using the Fourier transform and the exact global dynamic stiffness matrix in the wavenumber domain is established describing the relationship between the generalized displacement and force vectors. Then, solutions of the multi-layered system for discrete wavenumbers are obtained by using the dynamic stiffness matrix method, which are then synthesized to retrieve the responses in the physical domain. The accuracy of the method is confirmed by comparison with existing solutions for the TI elastic as well as isotropic saturated media that are special cases of the more general problems addressed. Selected numerical results are presented to investigate the effects of material anisotropy, layering, surface drainage condition, buried depth of the source and permeability on the responses. It is found that material anisotropy is very important for the accurate assessment of the dynamic responses subjected to time-harmonic sources. In addition, the presented solutions form a complete set of Green's functions which is required in the application of plane strain boundary methods for multi-layered TI saturated medium.

1. Introduction

The dynamic response of a semi-infinite medium due to surface or buried loads is a subject of considerable interest in applied mechanics and civil engineering because of its relevance to earthquake engineering, dynamics of foundations, and modeling of geophysical testing methods. Since the pioneering work of Lamb (1904) dealt with the tremors propagation over the surface of an elastic medium, a large number of authors have considered the problem of the dynamic responses of isotropic media. The excellent works of Achenbach (1973), Aki and Richards (1980), Apsel and Luco (1983), Banerjee and Mamoon (1990), Miklowitz (1984), Pak and Guzina (2002) and Kausel (2006) have dealt with a variety of elastodynamic problems related to isotropic elastic materials. In addition, based on the first theory of propagation of elastic waves in a fluid-saturated porous medium established by Biot (1956a,b, 1962), dynamic responses of a homogeneous poroelastic half-

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space due to time harmonic loads have been addressed by Paul (1976), Philippacopoulos (1988a,b, 1989, 1997), Jin and Liu (2001), Senjuntichai and Rajapakse (1994) and Zheng, Zhao, and Ding (2013a), and solutions for a multi-layered poroelastic half-space have been presented using the propagator matrix method (Lu & Hanyga, 2005; Zheng, Ding, Zhao, & Ding, 2013b) and the dynamic stiffness matrix method (Knopoff, 1964; Liang & You, 2004; Rajapakse & Senjuntichai, 1995).

Natural soils and rocks are rarely isotropic but anisotropic as evidenced by numerous experimental findings (Atkinson, 1975; Pickering, 1970). In the point of practical engineering, the anisotropic nature of a variety of soils and rocks can be represented by a reduced form of anisotropy known as transversely isotropic (TI). Regarding the dynamic response of a TI medium, Stoneley (1949) initially investigated the wave propagation in a TI medium. Later, Payton (1983) summarized the dynamic problems of a TI elastic half-space under surface loads. In the following years, especially in recent years, Rajapakse and Wang (1991, 1993) presented the two-dimensional (2-D) and three-dimensional (3-D) Green's functions for a homogeneous TI half-space due to interior time-harmonic loading. Liu, Li, and Sun (1997) investigated axisymmetric wave propagation problem of a TI half-plane solids. Wang and Liao (1999) presented the closed-form solutions for the displacements and stresses in a homogeneous TI half-space subjected to various buried loading types. Shodja and Eskandari (2007) addressed the solutions of a TI half-space with a coating TI layer under axisymmetric time-harmonic loadings. Khojasteh, Rahimian, Eskandari, and Pak (2008a), Khojasteh, Rahimian, Pak, and Eskandari (2008b) derived the asymmetric dynamic Green's functions of a homogeneous and a two-layered TI half-space with the aid of the potential functions (Rahimian, Eskandari-Ghadi, Pak, & Khojasteh, 2007). Khojasteh, Rahimain, Eskandari, and Pak (2011) extended their solutions to the multi-layered TI half-space using the transmission-reflection matrix method. Ai and Li (2014), Ai, Li, and Cang (2014) and Ai and Zhang (2015) proposed an analytical element method to address the axisymmetric, asymmetric and plane strain dynamic responses of a multi-layered TI half-space. Chen (2015) also studied the 3D dynamic Green's functions for a multi-layered TI half-space utilizing the Fourier-Bessel transform and precise integration method.

It should be noted that the above studies related to the TI medium are still restricted to the elastic or viscoelastic cases. However, in many cases, geomaterials are not only transversely isotropic but also fluid-saturated, especially in coastal areas, which are more reasonably be modeled as a TI saturated medium. Studies of propagation of elastic waves in a TI saturated material are rather limited. Kazi-Aoual, Bonnet, and Jouanna (1988) sought for a steady-state wave propagation solution for an infinite TI saturated solid using the Kupradze's method (Kupradze, 1979), however, the explicit forms of the solutions were not obtained there. Taguchi and Kurashige (2002) presented the full TI saturated space Green's functions for step-like point sources utilizing the Kupradze's method together with the Fourier and Hankel transforms, however, no numerical solutions are given in their study. Very recently, using the potential functions, Sahebkar and Eskandari-Ghadi (2016) obtained the dynamic response of a saturated porous TI half-space subjected to time-harmonic surface loads for the first time. To the best knowledge of the authors, dynamic solutions and a set of numerical results for a multi-layered TI saturated half-space due to dynamic sources has not been reported in the literature.

The objective of this paper is to propose a computationally stable and efficient exact stiffness matrix method for evaluation of the plane strain dynamic responses of a multi-layered TI saturated half-space subjected to time-harmonic surface or buried sources. The calculations are performed in the Cartesian coordinate system, and solutions in the frequency domain are presented for displacements (including displacement of fluid with respect to the solid), stresses and pore fluid pressure corresponding to horizontal and vertical loadings and pore fluid pressure. The time-harmonic sources can be either uniformly distributed or concentrated. As in practical engineering, strip foundations, embankments, dams and topographies such as canyon, alluvial valley and hills are generally considered as the plane strain problems, which is more suitable for Cartesian coordinates, the presented fundamental solutions are of great value in developing 2-D boundary value approaches to solve a variety of wave scattering, wave radiation and soil-structure interaction problems in a multi-layered TI saturated half-space. This paper is organized as follows: Section 2 describes the theoretical approach concerning specially the multi-layered TI saturated half-space. It includes establishing the exact global dynamic stiffness matrix and using the stiffness matrix method to obtain solutions in the frequency domain. Section 3 is devoted to the verification of the method. Section 4 is intended to parametric studies and discussions, in which the effects of material anisotropy, layering, surface drainage condition, buried depth of the source and permeability on the dynamic responses are analyzed. And finally, the conclusions are given in Section 5.

2. Model and theoretical formulations

The model of a multi-layered TI saturated half-space subjected to surface or buried time-harmonic sources is shown in Fig. 1. The time-harmonic sources include uniform strip or concentrated horizontal and vertical loadings and pore fluid pressure. The layered system is formed by arbitrary number of TI saturated layers ($1 \sim N$) and the underlying TI saturated half-space ($N+1$), which are all characterized by Boit's theory (Biot, 1956a,b, 1962). The layers are perfectly bonded with each other and have arbitrary thickness of d_n ($n = 1 \sim N$). The surface of the layered half-space is free from tractions and a radiation condition is imposed on the underlying half-space. Two drainage conditions on the free surface of the half-plane medium are considered in this study, which are defined as the drained condition (the surface is absolutely permeable) and the undrained condition (the surface is completely impermeable), respectively.

The exact dynamic stiffness matrix method is developed to solve the problem. First, the two-dimensional Biot's equations of **u**-p formulation are Fourier transformed into the wavenumber domain with respect to the x-coordinate. Using the potential function, the corresponding ordinary differential equations are solved to present the analytical solutions in the