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Heat-pulse propagation in functionally graded thin layers

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ABSTRACT

In the present paper we study how thermal pulses propagate in functionally graded thin layers. The present theoretical analysis lies on a heat-transport equation which goes beyond the classical Fourier law. The compatibility of that equation with the main tenets of continuum mechanics is investigated by means of second law of thermodynamics. Theoretical considerations are also made on the nonequilibrium temperature, the thermal-conductivity tensor and the form of the specific-entropy flux.

1. Introduction

Since 1822, when the French mathematician and physicist Jean Baptiste Joseph Fourier published his book about the analytic theory of heat (Fourier, 1878), in any engineering problem regarding the heat conduction it is prescribed that the local heat flux q_i be proportional to the gradient of the local-equilibrium temperature T, and it can only flow from the warmer regions to colder ones (Fourier, 1878). For macroscopic systems, such a statement is summarized by the following semi-linear constitutive equation

$$q_i = -\Lambda_{ij} T_{,j} \tag{1}$$

wherein $\Lambda_{ij} \equiv \Lambda_{ij}(T)$ is the temperature-dependent positive-definite symmetric second-order tensor of the thermal conductivities, which reduces to a scalar function in the case of isotropic materials. In literature it is possible to find several theoretical derivations of Eq. (1), which is also well-known as the Fourier law. Based on the kinetic theory of gases, the Boltzmann's proof is worth of mention for its clarity and expressivity (Boltzmann, 1902; Cattaneo, 1948).

Despite its empirical standing in ordinary circumstances, both theoretical and experimental evidences show that Eq. (1) no longer holds in several nonequilibrium situations (Jou, Casas-Vázquez, & Lebon, 2010; Lebon, Jou, & Casas-Vázquez, 2008; Müller & Ruggeri, 1998), as for example at nanoscale (Ferry & Goodnick, 2009; Tzou, 2014; Volz, 2010; Zhang, 2007). The consequent inapplicability of Eq. (1) to well-describe heat transport in those situations led to several generalizations of

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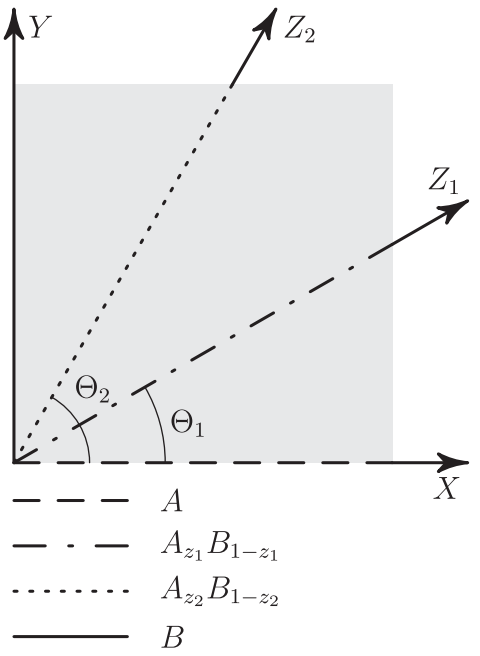

Fig. 1. Illustrative sketch of a two-dimensional functionally graded material of the type A_zB_{1-z}, with A and B two theoretical materials, and z as a stoichiometric variable which may vary in the interval [0, 1] whenever the in-plane direction is changed. For example, when the Z_1 in-plane direction is chosen (characterized by the Θ_1 degree in figure), the inner composition of the material along that direction is $A_{z_1}B_{1-z_1}$, with z_1 being a real number such that $0 < z_1 < 1$. Similarly, when Z_2 is another in-plane direction (characterized by the Θ_2 degree in figure), the inner composition of the material along that direction is $A_{z_2}B_{1-z_2}$, with $z_2 \neq z_1$ as a real number such that $0 < z_2 < 1$.

it (Chen, 2001; Ciarletta, Straughan, & Tibullo, 2012; Grmela, Lebon, Dauby, & Bousmina, 2005; Lebon, Machrafi, Grmela, & Dubois, 2011; Lebon, 2014; Sellitto, Cimmelli, & Jou, 2016; Ván & Fülöp, 2012). Note that in addition to referring to small systems whose characteristic length is of the order of nanometers, the appellation *nanosystems* also refers to those systems which are characterized by an internal nanostructure that gives them some special mechanical, thermal, electrical and optical features. Such structures may be, for example, nanopores or nanoparticles, several parallel or very thin layers (or films), and functionally graded materials (FGMs) (Birman & Byrd, 2007; Jou, Carlomagno, & Cimmelli, 2015; Li, Wu, Fan, Yang, & Majumdar, 2003; Machrafi & Lebon, 2015; Mansoor & Yilbas, 2016; Minakov & Schick, 2016; Sellitto, 2015).

FGMs are not a separate class of materials, but they rather represent an engineering approach to modify the structural and/or chemical arrangement of materials or elements. It is very difficult to provide broad design guidelines for the use of FGMs because their structures may be very complex and diverse, as well as their thermomechanical features still show some unexpected behaviors.

Much of the understanding about heat transfer in FGMs comes from computer numerical simulations which allow for detailed and accurate descriptions (Burlayenko, Altenbach, Sadowski, Dimitrova, & Bhaskar, 2017; Chen, Tong, Gu, Zhang, & Ochoa, 2004; Marin, 2005; Reddy, 2000; Sladek, Sladek, & Zhang, 2003). However, this should not make us forget the practical usefulness and the conceptual challenge of mesoscopic approaches which firstly start from a macroscopic perspective and then deepen into more detailed and accurate descriptions of the physical system. According with this point of view, for a better understanding of the use of FGMs, on pure theoretical grounds in this paper we investigate how thermal pulses are predicted to move once a heat-transport equation beyond Eq. (1) is introduced. Together with the well-known energy-balance equation, in the absence of thermal induced deformations and stresses, that heat-transport equation represents the basic law of our theoretical model which we propose here for the description of the thermal behavior of FGMs. We prove that the aforementioned model, developed in the framework of Extended Irreversible Thermodynamics (Jou, Casas-Vázquez, & Lebon, 2010; Lebon, Jou, & Casas-Vázquez, 2008), agrees with well-known tenets of continuum mechanics.

Our analysis may also have interesting consequences in practical applications; for example, it may provide a tool to understand how specific gradations in structure and/or composition affect the thermo-mechanical behavior of those materials.

In our thermal-pulse propagation analysis, in a first rough approximation, we put ourself in the two-dimensional case and sketch the FGM as a sheet. Then, for a chosen spatial direction of the plane, we further assume that the inner composition of the material at hand is of the type A_zB_{1-z}, with A and B two theoretical materials, and z as a stoichiometric variable which may change in the range [0, 1] with continuity whenever the aforementioned spatial direction changes its slope with respect to the longitudinal direction (see Fig. 1 for an illustrative sketch of the system we consider here). In our model, therefore, there is a one-to-one relation between z and Z: a given value of z indicates a precise spatial direction Z in the plane containing the FG sheet and, conversely, a given in-plane spatial direction indicates a precise value of the stoichiometric variable.