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Thermoelectric fields and associated thermal stresses for an inclined elliptic hole in thermoelectric materials

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ABSTRACT

A theoretical model for the thermoelectric coupling analysis of thermal materials with an inclined elliptic hole is constructed. The extended problem of an elliptic hole under biaxial loading is also discussed. Theoretical and numerical results for the thermoelectric coupling resulting thermal stress are obtained. Results show that, for the biaxial loaded elliptic hole, the solution cannot be the linear superposition of the solutions of two uniaxial loaded cases due to the nonlinear coupling. For inclined elliptic hole, the maximum thermoelectric concentration occurs when the major axis is perpendicular to the loading direction. However, the maximum stress concentration occurs when the major axis is parallel to the loading direction. The solution of a circle hole problem is given as a special case in the framework of elliptic hole problem. The crack problem is also discussed when the ellipse degenerates into a crack. It is found that all field intensity factors exhibit the traditional inverse square-root singularity at the crack tip. The mode I stress intensity factor only depends on the applied electric current. The mode II stress intensity factor only depends on the applied energy flux when the crack line is perpendicular to the loading direction. However, in the biaxial loading case, the mode II stress intensity factor relies on the applied electric current density and energy flux. This is the first paper to conduct a strict closed-form solution for an inclined elliptic hole in thermoelectric materials.

1. Introduction

Thermoelectric materials have wide engineering applications, such as for generating electricity from waste heat, solid state thermal management, carbon reduction and solar energy harvesting (Bell, 2008; Kraemer, Poudel, & Feng, 2011; Narducci, 2011; Yang & Caillat, 2006). Nowadays, about two-third of our used energy is lost as waste heat. There is an urgent need for high efficiency thermoelectric materials to directly convert waste heat into electric energy (Biswas, He, & Blum, 2012). The thermoelectric materials therefore have gained significant attentions in the field of materials science (Snyder & Toberer, 2008).

Better utilizing thermoelectric materials demands a better understanding of their mechanical properties. Considerable investigations have been carried out in stress analysis of thermoelectric materials. Jin (2013) studied the buckling behavior of thin film thermoelectrics subjected to a temperature gradient. Wang, Guo, and Zhang (2015) conducted a thermal shock

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resistance for a Bi_2Te_3 based thermoelectric material plate subjected to one-dimensional temperature load. Zhao, Zhang, and Li (2008) investigated the effects and mechanical properties of nano-SiC addition on the thermoelectric. Gao, Du, and Zhang (2011) presented the thermal stress analysis and optimized the structure parameters for a thermoelectric material. Jin (2014) investigated the thermally induced stresses in a multilayered thin film thermoelectric structure and assessed the thermomechanical reliability of the thermoelectric devices. Huang, Yen, and Wang (2005) studied the influence of Thomson effect on the cooling efficiency of a thermoelectric cooler and found that the cooling efficiency can be improved not only by increasing the figure of merit of the thermoelectric materials but also by taking advantage of the Thomson effect.

Although great progresses have been achieved for thermoelectric material development, theoretical work towards continuum analysis of thermoelectric materials is limited despite its key role in the development of thermoelectric materials. Wang, Huang, and Cheng (2012) proposed a general, three-dimensional numerical model for thermoelectric materials with consideration of coupling of temperature field and electric potential field. Liu (2012) developed a continuum theory for thermoelectric bodies following the frameworks of continuum mechanics and general principles of thermodynamics. Wang (2017) proposed a finite element model for the determination of time dependent thermoelectric coupling fields taking into account of all thermoelectric effects, including Joule heating, Thomson effect, Peltier effect and Fourier's heat conduction and temperature dependent material properties. Yang, Lei, Gao, and Li (2015) developed an asymptotic homogenization theory to analyze the effective behavior of three-dimensional thermoelectric composites.

On the other hand, thermoelectric materials are typical brittle solids. Defects or cracks produced in manufacturing or operation conditions may cause thermoelectric field concentration at the crack tip and reduce the strength of the materials. Understanding the thermoelectric fields and associated thermal stresses in thermoelectric materials with cavities is the key to figure out how the cavities affect the thermoelectric properties. Schmidt, Case, and Giles (2012) studied the mechanical properties and crack growing behavior of thermoelectric material Mg_2Si. Zhang and Wang (2016b) investigated the interface crack in a layered thermoelectric material subjected to thermoelectric loadings. Eilertsen, Subramanian, and Kruzic (2013) explored the fracture toughness of Co_4Sb_{12} and $In_{0.1}Co_4Sb_{12}$ thermoelectric skutterudites. The electric and heat conductions across a crack in a thermoelectric material are studied by Song, Song, Song, and Song (2016). The solutions for a crack in thermoelectric materials under bidirectional loading are presented by Song, Gao, and Li (2015) Due to the randomness of the generated crack in thermoelectric materials, the inclined crack problem should be a research focus. Sosa (1991) derived the general solution of a two-dimensional transversely isotropic piezoelectric material containing defects. Tsukrov and Kachanov (2000) investigated a two-dimensional anisotropic solid with elliptical holes having an arbitrary (non-random) orientational distribution and the effective moduli are given. Li, Jiang, Li, Yang, and Zhang (2016) discussed a finite plane contains an inclined crack subjected to biaxial tensile loads. Qin and Mai (1997) determined the crack propagation in the case of an inclined crack in a half-plane thermopiezoelectric sheet subjected to uniform heat flow by using the strain energy density theory. Sharma, Bui, Bhargava, Yu, Lei, and Hirose (2016) analyzed an array of equidistant inclined cracks in two-dimensional piezoelectric strip using distributed dislocation method. Although the inclined crack problems have been widely studied, the inclined crack problem for thermoelectric materials has never been presented. It is noteworthy that Zhang and Wang (2016a) derived the solution for an orthogonal elliptic hole in thermoelectric materials. In engineering practice, the inclusions are randomly distributed. Thus, it is essential to solve the inclined inclusion problem.

In the present work, closed-form solution for the two-dimensional problem of an inclined elliptic hole in thermoelectric materials is obtained. The solution of an elliptic hole under biaxial loading is also obtained. The paper is organized as follows: First, governing equations of electric current density, temperature and stress fields and associated boundary conditions are derived; next, solution of electric current density, temperature, and stress field are derived in closed-form using complex variable technique; the solutions for some special cases are given subsequently: the circle hole problem is solved, the crack problem is discussed by degenerating the ellipse into a flat crack; finally, some numerical examples are given and concluding remarks are made.

2. Governing equations and their solutions

In the stationary case when no free electric charge and heat source exist, the basic equations for a homogeneous thermoelectric material can be written as: (Perez-Aparicio, Taylor, & Gavela, 2007)

(i) Thermoelectric constitutive laws:

$$\mathbf{j} = -\sigma\nabla V - \sigma\varepsilon\nabla T,\ \mathbf{q} = -\lambda\nabla T + \varepsilon T\mathbf{j}, \tag{1}$$

where $\mathbf{j}$ is electric current density and $\mathbf{q}$ is heat flux, V, T, ε, σ, λ are electric potential, temperature, Seebeck coefficient, electric conductivity and thermal conductivity, respectively.

(ii) Thermoelectric equilibrium equations:

$$\nabla\cdot\mathbf{j} = 0,\ \nabla\cdot\mathbf{e} = 0, \tag{2}$$

where $\mathbf{e}$ is energy flux defined as $\mathbf{e} = \mathbf{q} + \mathbf{j}V$. These equations reflect the electric charge equilibrium and energy equilibrium of an enclosed volume.

From these equations we can see that electric current density consists two parts, where $-\sigma\nabla V$ represents the Ohm electric current and $-\sigma\varepsilon\nabla T$ represents the electric current produced by the Seebeck effect. In addition, the heat flux produced