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Effective in-plane elastic moduli of quasi-random spatially irregular hexagonal lattices

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ABSTRACT

An analytical framework is developed for predicting the effective in-plane elastic moduli (longitudinal and transverse Young's modulus, Poisson's ratios and shear modulus) of irregular hexagonal lattices with generalized form of spatially random structural geometry. On the basis of a mechanics based bottom-up multi-step approach, computationally efficient closed-form formulae are derived in this paper. As a special case when there is no irregularity, the derived analytical expressions reduce to the respective well known formulae of regular honeycombs available in literature. Previous analytical investigations include the derivation of effective in-plane elastic moduli for hexagonal lattices with spatially random variation of cell angles, which is a special case of the generalized form of irregularity in material and structural attributes considered in this paper. The present study also includes development of a highly generalized finite element code for obtaining equivalent elastic properties of random lattices, which is employed to validate the proposed analytical formulae. The statistical results of elastic moduli obtained using the developed analytical expressions and using direct finite element simulations are noticed to be in good agreement affirming the accuracy and validity of the proposed analytical framework. All the in-plane elastic moduli are found to be significantly influenced by spatially random irregularity resulting in a decrease of the mean values for the two Young's moduli and two Poisson's ratios, while an increase of the mean value for the shear modulus.

1. Introduction

Hexagonal lattices/lattice-like structural forms are present as materials and structures in abundance across various length-scales (nano, micro and macro) within natural systems and artificial products, as shown in Fig. 1 (Gibson & Ashby, 1999). Such structures have received considerable attention in last few decades as an advanced material because of the capability to meet high performance application-specific demands in various critically desirable parameters such as specific strength and stiffness, crushing resistance, fatigue strength, acoustic properties, shock absorption properties, electro-mechanical properties, corrosion and fire resistance (Gibson & Ashby, 1999). The application of honeycomb cores for lightweight sandwich structures is an active area of research (Mukhopadhyay & Adhikari, 2016c; Yongqiang & Zhiqiang, 2008; Zenkert, 1995). Honeycomb grill is commonly used to reduce noise and facilitate smooth airflow in computer fans. An in-depth understanding of the structural behavior of such hexagonal lattices is useful in emerging research fields of

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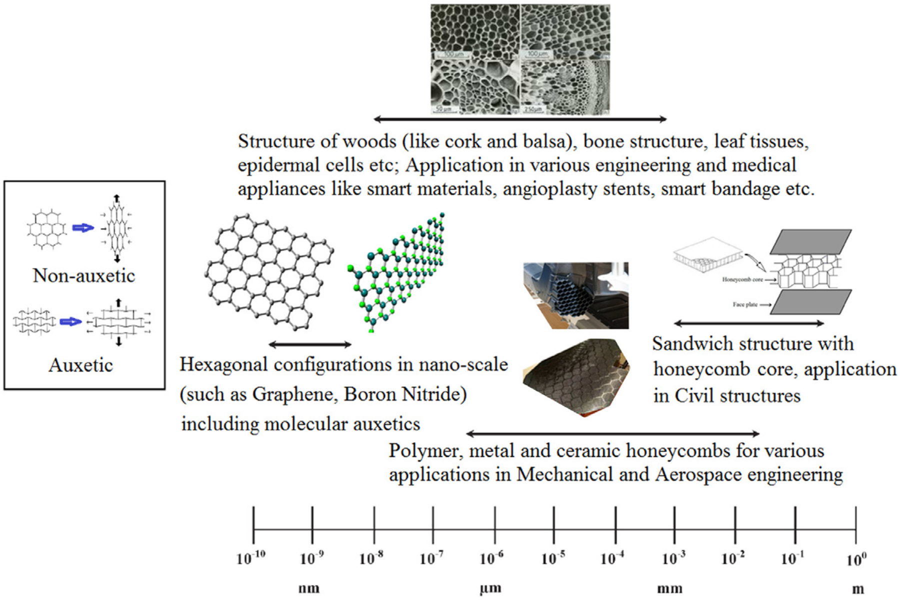

Fig. 1. Occurrence and application of hexagonal lattices across the length scales.

nano-materials like Graphene and Boron Nitride, which are often idealized as hexagonal periodic structures (Liu, Xie, Zhang, Zheng, & Xu, 2012; Mukhopadhyay, Mahata, Adhikari, & Zaeem, 2017; Pantano, Parks, & Boyce, 2004).

To eliminate the need of a detailed finite element modelling for hexagonal lattices/honeycombs as a part of another complex structural system (host structure such as a sandwich panel), such lattices are generally modeled as a continuous solid medium with equivalent elastic moduli throughout the domain. For example, the effective elastic properties of the honeycomb-core are required to characterize the static and dynamic response of the sandwich panels such as deflection, natural frequency etc. Estimation of effective elastic properties is quite common in the literature of mechanical sciences (Michel, Moulinec, & Suquet, 1999; Tang & Felicelli, 2015; Vilchevskaya & Sevostianov, 2015). A similar approach is followed to evaluate the effective material properties of different nano-structures having hexagonal configurations (Mukhopadhyay, Mahata, Adhikari, & Zaeem, 2017). It is a common practice to consider a representative unit cell to model various other periodic structures (Javid et al., 2016). Extensive research has been conducted so far to predict effective elastic properties of regular hexagonal lattices without any form of irregularity (El-Sayed, Jones, & Burgess, 1979; Gibson & Ashby, 1999; Goswami, 2006; Malek & Gibson, 2015; Zhang & Ashby, 1992). Other crucial research areas concerning different responses related to honeycombs include crushing behavior, low velocity impact, buckling analysis and wave propagation through lattices (Gonella & Ruzzene, 2008a; 2008b; Hu & Yu, 2013; Jang & Kyriakides, 2015; Jimenez & Triantafyllidis, 2013; Klintworth & Stronge, 1988; Liu et al., 2016; Schaeffer & Ruzzene, 2015; Wilbert, Jang, Kyriakides, & Floccari, 2011; Zschernack, Wadee, & Vollmecke, 2016). A substantial amount of scientific literature is available dealing with perfectly periodic hexagonal auxetic lattices (Critchley et al., 2013; Evans & Alderson, 2000). Recently theoretical formulations have been presented for equivalent elastic properties of periodic asymmetrical honeycomb (Chen & Yang, 2011). Tailorable elastic properties of hierarchical honeycombs and spiderweb honeycombs have also been reported (Ajdari, Jahromi, Papadopoulos, Nayeb-Hashemi, & Vaziri, 2012; Mousanezhad et al., 2015). Analysis of two dimensional hexagonal lattices/honeycombs, as presented in the above literature review, are based on an unit cell approach, which can be applied only for perfectly periodic lattice forms.

The major limitation of the aforementioned unit cell based approach is that it cannot be used to analyze a system with spatial irregularity. Spatial irregularity/variability in lattices is practically inevitable; it may occur due to structural defects, manufacturing uncertainty, variation in temperature, micro-structural variability and pre-stressing. Moreover, development of novel metamaterials (Mukhopadhyay and Adhikari, 2017; Srivastava, 2016) having hexagonal micro-structures may involve spatially varying structural and material attributes. To consider the effect of irregularity in cellular lattices, voronoi honeycombs are found to be considered in the literature (Li, Gao, & Subhash, 2005; Zhu, Hobdell, Miller, & Windle, 2001; Zhu, Thorpe, & Windle, 2006). Dynamic crushing of honeycombs with irregularity in cell wall thickness and cell shapes have been investigated (Li, Gao, & Wang, 2007). Triantafyllidis and Schraad (1998) have studied the failure surface of aluminium honeycombs for general inplane loading considering micro-structural imperfections. Papka and Kyriakides (1994); 1998) and Jang and Kyriakides (2015) have reported numerical and experimental study of honeycomb crushing and buckling behavior accounting geometrical imperfections, such as over/under expanded cells and variation in length of bond line. The effect due to defects on regular as well as voronoi honeycombs and the effect of manufacturing uncertainty on auxetic honeycomb have been reported by Ajdari, Nayeb-Hashemi, Canavan, and Warner (2008) and Liu, Wang, Huang, and Zhong (2014), respectively. Though the above mentioned studies substantially investigate the effect of irregularities based on lim-