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On the mechanics of laminated microplates

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ABSTRACT

This paper investigates the mechanical behavior of micro scaled laminated composite plates based on the modified couple stress theory. To this aim, considering a displacement field for the plate, the kinematic parameters such as strains and curvatures are calculated. Utilizing these kinematic parameters, classical stresses and couple stresses are obtained in each layer and consequently the equation of motion is derived using equilibrium equations of forces and moment of forces. Equivalent mechanical properties including the length scale parameter are presented for the composite plate. Using these equivalent quantities, static deflection and natural frequency of bilayer microplates are investigated and effects of size dependency on the mechanical behavior of the structure are assessed.

1. Introduction

Microplates can be considered as important parts of various microsystems such as micropumps, microresonators, mass sensors and other micro-electro-mechanical systems. Since, the static and dynamic behavior of the microplate affects the functionality of the abovementioned microsystems, many researchers have studied the mechanical behavior of these structures (Ansari, & Gholami, 2016; Arani, & Jafari, 2015; Batra, Porfiri, & Spinello, 2008; Chao, Chiu, & Tsai, 2006; Farahmand, Ahmadi, & Arabnejad, 2013; Guo, Chen, & Pan, 2016; Li, & Pan, 2015; Li, Packirisamy, & Bhat, 2008; Moghimi Zand, & Ahmadian, 2007; Pradhan, & Phadikar, 2011). As an example, Zhao, Abdel-Rahman, & Nayfeh, (2004) utilized the finite element method to present a model for static and dynamic behavior of microplates. They used this model to investigate the static deflection and natural frequency of electrostatically actuated microplates. Wang, Lin, Li, & Feng, (2011) modeled the mechanical behavior of pre-stressed circular microplates under electrostatic actuation and investigated the static deflection and free vibration of the structure. Li, Packirisamy, & Bhat, (2008) used a reduced order model to analyze the static and dynamic behavior of microplates used in micromirrors and compared their results with those predicted by the finite element method.

Not only homogenous microplates, but also laminated composite microplates are widely used in microsystems and hence researchers have investigated the mechanical behavior of these components. Arani, & Jafari, (2015) investigated the vibration behavior of laminated microplates resting on an elastic foundation. They used Hamilton's principle to derive equations of motion and solved these equations by utilizing differential quadrature method. Zand, & Ahmadian, (2007) presented a hybrid finite element/finite difference method to model the dynamic behavior of electrostatically actuated laminated microplates. Zuo, Liu, & Li, (2013) studied the thermomechanical behavior of laminated circular microplates and presented a model for thermoelastic damping in composite plate resonators. Changizi, Stiharu, Olbrechts, & Raskin, (2015) proposed an analytical model to predict the residual stress in laminated microplates. They used the static deflection of the structure to estimate the residual stress and compared their results with experimental observations.

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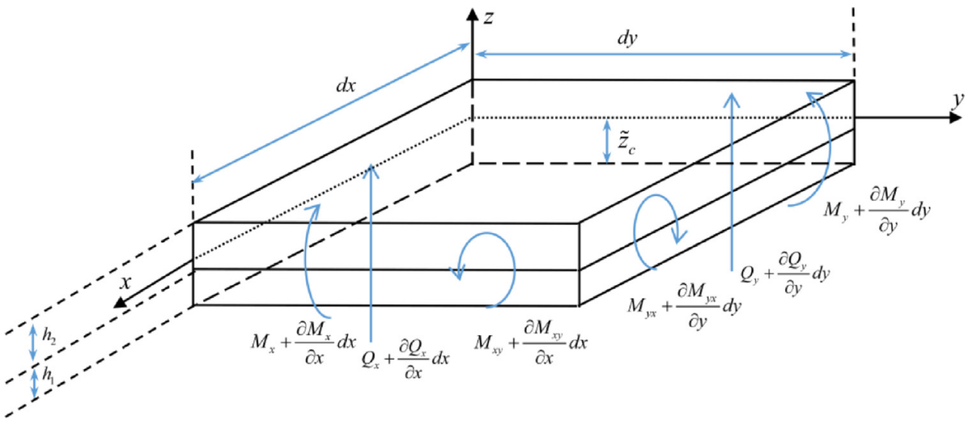

Fig. 1. A bilayer microplate-Geometry and coordinate system.

The experimental observations performed on the mechanical behavior of small structure proved that the stiffness of these components are significantly higher than the stiffness expected based on the classical continuum theory (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; Liu, Ke, Wang, Yang, & Kitipornchai, 2014; McFarland, & Colton, 2005; Stölken, & Evans, 1998). It is noted that this phenomenon is called size dependency in the literature. According to these experiments, it was concluded that the classical theory neglects a structural phenomenon which can be important at micro scale although it is negligible at macro scale. Due to inadequacy of the classical theory, new nonclassical continuum theories such as couple stress theory and modified couple stress theory were proposed. Yang, Chong, Lam, & Tong, (2002) introduced the modified couple stress theory by performing a modification on the couple stress theory formulated by Mindlin, & Tiersten, (1962). In the modified couple stress theory, an element of material is considered to be subjected to couple stresses in addition to classical normal and shear stresses. These couple stresses are related to the rotation gradient by a new material constant known as length scale parameter. Comparing the results of the modified couple stress theory with experimental observations indicated that in contrast to the classical theory, this theory can accurately model and predict the mechanical behavior of micro scale structures . Due to the capability of the modified couple stress theory in modeling of micro scale structures, this theory has become very popular and many researchers have used it to investigate mechanical behavior of small structures including microbeams and microplates (Hu, Wang, Dai, Wang, & Qian, 2016; Mohammad-Abadi, & Daneshmehr, 2015; Mojahedi, 2017; Mojahedi, & Rahaeifard, 2016). Kong, Zhou, Nie, & Wang, (2008) utilized this theory to derive the governing equation of Euler-Bernoulli microbeams. Ma, Gao, & Reddy, (2008) developed a size dependent model to investigate the static and dynamic behavior of Timoshenko microbeams. Size dependent mechanical behavior of Kirchhoff plates with arbitrary shapes is studied by Tsiatas (Tsiatas, 2009). Yin, Qian, Wang, & Xia, (2010) investigated the dynamic behavior of microplates based on the modified couple stress theory. They presented natural frequency of the plate and compared their results with the results of the classical theory to show the effects of size dependency on the dynamic response of the system. Rahaeifard, & Mojahedi, (2016) studied the dynamic behavior of electrostatically microaccelerometers based on the modified couple stress theory and investigated the effects of size dependency on the dynamic pull-in and nonlinear vibration of the device. Furthermore, this theory is utilized by researchers to analyze the mechanical behavior of microbars (Tsiatas, & Katsikadelis, 2011), composite microbeams (Mohammadabadi, Daneshmehr, & Homayounfard, 2015; Rahaeifard, 2016) and the micro scale structures made of functionally graded materials (Akbaş, 2016; Ke, Wang, Yang, & Kitipornchai, 2012; Li, & Pan, 2015; Şimşek, & Reddy, 2013).

Due to the wide application of laminated composite plates in microsystems and the accuracy of the modified couple stress theory in modeling the small structures, in this paper the governing equation of a bilayer microplate is derived based on the modified couple stress theory. Considering the displacement field for the plate, kinematic parameters and consequently classical stresses and couple stresses are calculated. Using these quantities and utilizing the equilibrium equations, the governing equation of motion is derived and solved using Galerkin's method. Equivalent material properties are defined for the laminated plate and the static deflection and natural frequency of the plate are studied using these equivalent properties. Results indicate that for the microplates with the thickness in order of the equivalent length scale, the size-dependency significantly affects the static and dynamic responses of the system.

2. Modeling of the bilayer microplate

Consider a small element of a laminated microplate of thickness *h* as shown in Fig. 1.

It is assumed that the plate is under transverse distributed load in *z* direction denoted by $f(x,y,t)$. The internal forces and couples caused by stresses and couple stresses in *x* and *y* planes shown in the figure (the opposite forces and couples in the back faces are eliminated to make the figure clear). The thicknesses of layers are represented by h_1 and h_2 (i.e. $h = h_1 + h_2$). The origin of coordinate system is located at the corner of neutral surface of the structure which is at the distance of $\tilde{z}_c$