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Calculation of the effective thermal conductivity of multiscale ordered arrays based on reiterated homogenization theory and analytical formulae

Eduardo S. Nascimento[a], Manuel E. Cruz[a,*], Julián Bravo-Castillero[b,1]

[a] UFRJ-Federal University of Rio de Janeiro PEM/COPPE, CP 68503 Rio de Janeiro, RJ 21941-972, Brazil

[b] Universidad de La Habana, Departamento de Matemática Facultad de Matemática y Computación San Lázaro y L, Habana 4, La Habana 10400 Cuba

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ABSTRACT

In this paper the effective thermal conductivities of multiscale heterogeneous media with ordered microstructures are determined based on the reiterated homogenization method and analytical formulae available in the literature. While conventional homogenization has been extensively applied to thermal problems in two-scale media, reiterated homogenization appears to have been used, to date, mostly to formulate problems in heterogeneous media with more than two scales, rather than to calculate effective properties. Here, specifically, analytical formulae for the effective conductivities of the 2-D square array of circular cylinders and the 3-D simple cubic array of spheres are used, in conjunction with the appropriate reiterated homogenization expressions, to calculate the effective conductivities of the corresponding three-scale arrays of circular cylinders and spheres. The case with a perfect thermal contact at the interface is considered. The results for the effective thermal conductivity gain of each three-scale array relative to the two-scale counterpart are given in terms of the problem volume fractions and phase contrast. For each three-scale array the optimal volume fraction at the smallest structural scale that maximizes the conductivity gain is determined, as well as the optimal global volume fraction. In special, gains in excess of 9% may be achieved. The present approach thus allows for the systematic evaluation of conductivity gains solely on the basis of Fourier heat conduction and microstructural information.

1. Introduction

The determination of the effective properties of heterogeneous media have challenged scientists, mathematicians, and engineers for quite a long time. One possible approach to the problem is the now well-established method of homogenization, which is an up-scaling procedure that unravels the macroscopic behavior of a continuous medium from a consideration of the first principles applied to its smallest scales. For two-scale media, conventional homogenization of the linear elliptic partial differential equation is presented in Bakhvalov and Panasenko (1989), Bensoussan, Lions, and Papanicolaou (1978),

* Corresponding author.
E-mail addresses: eduardosnascimento@poli.ufrj.br (E.S. Nascimento), manuel@mecanica.coppe.ufrj.br (M.E. Cruz).

[1] Present address: Universidad Nacional Autónoma de México Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas Delegación Álvaro Obregón Apartado Postal 20–726, 01000 México, D.F. México.

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Nomenclature

d	smallest length scale of medium (m)
F^{G}	analytical formula available for array geometry G
k	thermal conductivity (W/m · K)
k_{eff}	effective thermal conductivity (W/m · K)
k^{*}_{gain}	effective thermal conductivity gain
k^{1}	intermediate effective thermal conductivity (W/m · K)
$\hat{k}$	global effective thermal conductivity (W/m · K)
L	largest length scale of medium (m)
V, $\|V\|$	domain, volume of domain (m^3)
$\mathbf{x}$	slow spatial coordinate (m)
X	generic cell, cell domain
$\mathbf{y}$	fast (Y-level) spatial coordinate (m)
Y	generic cell, cell domain, or dimension at the structural level δ
$\mathbf{z}$	fastest (Z-level) spatial coordinate (m)
Z	generic cell, cell domain, or dimension at the structural level d

Greek symbols

α	ratio of phase conductivities or phase contrast
Γ_Y	union of interfaces in the Y-level cells
Γ_Z	union of interfaces in the Z-level cells
δ	intermediate length scale of medium (m)
ϵ	small parameter, ratio of two successive length scales
ϕ	volume fraction
Ω	multiscale medium domain
$\partial\Omega$	external boundary of Ω

Subscripts

eff	effective
m	continuous or matrix component
opt	optimal
p	dispersed, inclusion or particle component

Superscripts

$*$	nondimensional quantity
CB	simple cubic array
CH	two-scale medium tractable by conventional homogenization
RH	three-scale medium tractable by reiterated homogenization
SQ	square array
Y	refers to the Y-cell or Y-domain at scale δ
Z	refers to the Z-cell or Z-domain at scale d

Cioranescu and Donato (1999) and Panasenko (2008). Heat transfer engineering applications of conventional homogenization, including numerical computations of the effective thermal conductivity, are found in Auriault (1983), Auriault and Ene (1994), Kamiński (2003), Matine, Boyard, Cartraud, Legrain, and Jarny (2013), Matt and Cruz (2008) and Rocha and Cruz (2001). For media with more than two spatial scales, the method of reiterated homogenization has been developed in Bensoussan et al. (1978), and used to formulate the linear elliptic boundary value problem (Allaire & Briane, 1996; Bensoussan et al., 1978), including heat conduction under perfect contact conditions at the interface (Rodríguez, Cruz, & Castillero, 2016). However, few effective property calculations, if any, have been performed to date based on the reiterated method.

Many current engineering materials or media exhibit heterogeneous structures at several different microscopic levels (Markov, Mousatov, Kazatchenko, & Markova, 2014; Telega, Gałka, & Tokarzewski, 1999), notably nanomaterials (Angayarkanni & Philip, 2015; Evans et al., 2008; Greco, 2014; Jin & Lee, 2013; Mortazavi, Benzerara, Meyer, Bardon, & Ahzi, 2013; Shin, Yang, Chang, Yu, & Cho, 2013; Vatani, Woodfield, & Dao, 2015; Wang, Zheng, Gao, & Chen, 2012). While it is expected that the macroscopic conductive behavior of such materials will be affected by these structures, few rigorous first-principle analytical or computational treatments are found addressing this issue. In the present paper, analytical calculations are performed of the effective thermal conductivity gain of multiscale ordered heterogeneous media based on reiterated homogenization theory.

Conflicting experimental reports on measurements of the effective thermal conductivity of colloidal nanofluids (Angayarkanni & Philip, 2015; Wang et al., 2012) point to the importance of complementary theoretical modeling approaches.