International Journal of Engineering Science 119 (2017) 217–231

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

On vibrations of nonlocal rods: Boundary conditions, exact solutions and their asymptotics

Xiao-Jian Xu, Mu-Lian Zheng*, Xuan-Cang Wang

Key Laboratory for Special Area Highway Engineering of Ministry of Education, School of Highway, Chang'an University, Xi'an 710064, PR China

ARTICLE INFO

Article history:
Received 28 March 2017
Revised 9 June 2017
Accepted 10 June 2017

Keywords:
Nonlocal strain gradient theory
Vibration
Boundary condition
Weighted residual method

ABSTRACT

The size effects on the dynamic behaviors of rods are investigated within the framework of the nonlocal strain gradient elastic theory. The variationally consistent boundary conditions are derived by using the weighted residual method with respect to the known equation of motion of rods. Similar to the fourth-order differential equation of motion for classical Euler–Bernoulli beams, the variationally consistent boundary conditions of nonlocal strain gradient rods are clarified for the first time. The exact characteristic equations for determining the longitudinal frequencies are presented for four types of boundary value problems. To gain insight into the asymptotic behaviors of rods, the reduced boundary values problems are also analytically given. The numerical results show that both the softening effect and the stiffening effect can be captured by adjusting the two material length parameters. However, when the two material length parameters are the same, the present results cannot recover to the corresponding classical ones for rods with general boundary conditions, which are different from the previous findings reported in the literature.

1. Introduction

Structures in terms of rods, beams, plates and shells are widely used in the engineering science. With the development of nanotechnology, these structures have been served as the key components in micro-electro-mechanical systems and nano-electro-mechanical systems (Eom, Park, Yoon, & Kwon, 2011). In order to well understand their mechanical behaviors, increasing interest in modeling the size effects of materials and structures at small length scales has given birth to the rapid development and application of the nonclassical continuum theories including but not limit to the strain gradient theory (Mindlin & Eshel, 1968; Toupin, 1964; Yang, Chong, Lam, & Tong, 2002) and nonlocal theory (Eringen, 1983).

In the context of strain gradient theory, it is assumed that the materials at small length scales should be modeled as atoms with higher-order deformation mechanism rather than modeled as collections of points. Based on this conceptual idea, significant contributions may be found in a number of works by Mindlin and Tiersten (1962), Toupin (1962), and recently by Yang et al. (2002) and Lam, Yang, Chong, Wang, and Tong (2003). Recent advances demonstrate that the strain gradient theory and its simplified version exhibit the following features:

(i) The differential order of equation of motion is increased due to the introduction of the higher-order strain gradient in the constitutive equation (Askes & Aifantis, 2011).

* Corresponding author.
E-mail address: zhengml@chd.edu.cn (M.-L. Zheng).

http://dx.doi.org/10.1016/j.ijengsci.2017.06.025
0020-7225/

(ii) The boundary conditions of the system are complex such that they should be correctly derived by the variational principles (Gao & Park, 2007; Tahaei Yaghoubi, Mousavi, & Paavola, 2017).
(iii) The solutions of the boundary value problems should be dealt with in due care so as to obtain the correct results (Polizzotto, 2013).
(iv) The materials exhibit stiffening behaviors compared with the classical continuum theory. It is concluded from the first three features that the explicit determination of the static and dynamic behaviors of materials is challenging. Although the mathematical difficulties will be encountered in analyzing the mechanical behaviors, this theory can only explain the stiffening effect observed in the experiments (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam et al., 2003; Sun, Han, Wang, & Lim, 2008).

On the other hand, the nonlocal theory assumes that the stress at a reference point in the body depends not only upon the strains at this point but also upon on strains at all other points of the body (Eringen, 1983). Recently, a variety of nonlocal models have been developed and successfully applied in the study of nanostructures such as carbon nanotubes and graphene sheets (Arash & Wang, 2012; Eltaher, Khater, & Emam, 2016; Rafii-Tabar et al., 2016). These works demonstrate, in general, that the materials exhibit softening behaviors compared with the corresponding classical continuum models. However, two issues should be addressed for nonlocal models:

(i) The nonlocal parameter is not included in the static bending problems for the cases of beams subjected to the concentrated loads (Li, Yao, Chen, & Li, 2015; Peddieson, Buchanan, & McNitt, 2003; Reddy & Pang, 2008).
(ii) The fundamental frequencies of the structures increase with increasing nonlocal parameters, and when the nonlocal parameters exceed to some values, the higher mode frequencies vanish (Li & Wang, 2009; Lu, Lee, Lu, & Zhang, 2007; Wang et al., 2007).

The first issue motivates Lim, Challamel and other researchers to improve the results by introducing the nonlocal parameter into the bending solutions. Along this line, one can refer to Lim (2007), Challamel and Wang (2008), Fernández-Sáez, Zaera, Loya, and Reddy (2016), Yan, Tong, Li, Zhu, and Wang (2015), Khodabakhshi and Reddy (2015), Barretta, Feo, Luciano, and Marotti de Sciarra (2015), Romano and Barretta (2016), Li, Yao, Chen, and Li (2015), Tuna and Kirca (2016) and Eptaimeros, Koutsoumaris, and Tsamasphyros (2016) for more details. These works reveal that the first issue may be solved by using some modifications of the constitutive equation, balance equation rather than directly applying the differential constitutive equation into the bending analysis. Recently, Romano, Barretta, Diaco, and Sciarra (2017) claimed that paradoxical results, encountered in the static boundary value problems of beams, stemmed from incompatibility between the constitutive boundary conditions and balance equations.

For the second issue, Xu, Deng, Zhang, and Xu (2016) presented the variationally consistent boundary value problems for dynamic behaviors of nonlocal differential Euler–Bernoulli and Timoshenko cantilever beams. In their work, they modified the shear force and bending moment by using the weighted residual method. Independently, Tuna and Kirca (2016) derived the exact solutions of boundary value problems of Eringen's nonlocal integral model. The common feature of the two works is that the dynamic softening effect can be captured for cantilever beams. The solved issues complementary to the other boundary value problems of nonlocal models demonstrate that the nonlocal theory results in the softening effect that can characterize the results of atom simulations (Arash & Wang, 2012; Li, Shen, & Lee, 2016; Shaat & Abdelkefi, 2017; Sundararaghavan & Waas, 2011; Zhang, Wang, Duan, Xiang, & Zong, 2009).

The above literature review shows that the strain gradient theory results in the stiffening effect rather than the softening effect predicted by the nonlocal theory. However, for materials at small length scales, both the stiffening effect and softening effect may be observed (Askes & Aifantis, 2009; Lim, Zhang, & Reddy, 2015; Xu & Deng, 2015). The longitudinal wave characteristics of rods were investigated by Güven (2014) and Shen, Wu, Song, Li, and Lee (2012) using the unified nonlocal elasticity models with two material length parameters. However, the effects of different boundary conditions on the wave characteristics were not reported. With regard to this fact, the combined nonlocal strain gradient theory established by Lim et al. (2015) are proposed by introducing both the nonlocal parameter and the strain gradient parameter in the constitutive equation. Owing to the fact that the nonlocal strain gradient theory is the general form of both nonlocal theory and strain gradient theory, the issues discussed above inherit naturally. However, these issues have not called into question by Li and Hu (2015), Li, Hu, and Li (2016), Li, Hu, and Ling (2015), Şimşek (2016), Farajpour, Yazdi, Rastgoo, and Mohammadi (2016), Ebrahimi, Barati, and Dabbagh (2016) and other researchers (Barati & Zenkour, 2017; Li et al., 2017). Recently, by using the weighted residual method, the exact bending and buckling solutions for complete boundary value problems of nonlocal strain gradient beams have been analytically obtained by Xu, Wang, Zheng, and Ma (2017). In contract to the conclusions given by Li and Hu (2015), Li, Hu, and Ling (2015), Li, Hu, and Li (2016), Şimşek (2016), Farajpour et al. (2016) and Ebrahimi et al. (2016), it is found, for the general boundary value problems, that the numerical results will not reduce to those of the corresponding classical ones for the same values of the two material length parameters. A direct evidence is that there may exist two alternative higher-order boundary conditions for structures with the prescribed boundary conditions; see the discussion by Xu et al. (2017).

In the present paper, the complete boundary value problems of the longitudinal vibrations of nonlocal strain gradient rods are studied in order to reach the following objectives. First, we derive for the first time the (lower-order and higher-order) variationally consistent boundary conditions for nonlocal strain gradient rods by utilizing the weighted residual method. Second, we identify and solve the corresponding boundary value problems by using the similarity between the