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Vibrations of Bernoulli-Euler beams using the two-phase nonlocal elasticity theory

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ABSTRACT

In this work the problem of the in-plane free vibrations (axial and bending) of a Bernoulli–Euler nanobeam using the mixed local/nonlocal Eringen elasticity theory is studied. The natural frequencies of vibration have been analytically obtained solving two uncoupled integro-differential eigenvalue problems, which are properly transformed in differential eigenvalue problems. Different kinds of end supports have been considered, and the influence of both mixture parameter and length scale has been analysed. The results show the softening effect of the Eringen's nonlocality, which is more pronounced as the local phase fraction decreases.

A large number of papers devoted to the dynamics of Bernoulli–Euler beams considering the fully nonlocal Eringen elasticity theory has been previously published. However, as recently stated by Romano, Barretta, Diaco and de Sciarra (2017), the problem is ill-posed in general, and the existence of a solution is an exception, the rule being non-existence. Nevertheless, the presence of a local term in the constitutive equation, leading to the two-phase formulation, renders the problem well-posed. To the best knowledge of the authors, this is the first time an exact solution is presented for a dynamic problem involving structures with constitutive equations corresponding to nonlocal integral Eringen's elasticity.

1. Introduction

The explosive growth of the nanotechnology and of the applications in the field of nanostructures has soared the studies related to nonlocal elasticity theories, among other generalized continuum mechanics approaches. The reasons are: (i) the classical elasticity is a scale-free theory which cannot adequately address the size effect commonly present in nanotechnology applications, and (ii) they are an attractive alternative to the huge computational cost of the Molecular Dynamic techniques.

The origin of nonlocal continuum mechanics theories can be found in papers by Kröner (1967), Krumhansl (1968) and Kunin (1968). Later, Eringen and coworkers (Eringen, 2002, 1972a, 1972b, 1983; Eringen & Edelen, 1972) simplified the above theories for linear homogeneous isotropic nonlocal elastic materials. Further, Eringen proposed a two-phase nonlocal model (Eringen, 1972a, 1987) which combine the classical local and the nonlocal constitutive theories. The basic feature of the Eringen theory of elasticity is that the stress at each point is related to the strain at all points in the domain. This influence decreases as the distance between the point of interest and the neighboring points increases.

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The nonlocal approach enabled different authors (Eringen, 1977; Eringen, Speziale, & Kim, 1977; Zhou, Bai, & Zhang, 1999) to address problems related with stress singularities which arise in classical fracture mechanics formulations, showing that these disappear using the nonlocal treatment. Additionally, these theories could overcome the difficulties showed by the classical local approaches to correctly predict solutions for problems in which microstructural and size effects are significant. These effects are present in modern engineering applications such as nano-machines (Bourlon, Glattli, Miko, Forro, & Bachtold, 2004; Drexler, 1992; Fennimore et al., 2003; Han, Globus, Jaffe, & Deardorff, 1997; Kim, Guo, Xu, & D.L., 2015), micro- or nano-electromechanical (MEMS or NEMS) devices (Arndt, Colinet, Arcamone, & Juillard, 2011; Berman & Krim, 2013; Ekinci & Roukes, 2005; Martin, 1996), or in biotechnology and biomedical areas (Bhushan, 2007; Saji, Choe, & Young, 2010).

The constitutive relation for the two-phase constitutive model originally proposed by Eringen (1972a, 1987) has been recovered by other authors (Altan, 1989; Benvenuti & Simone, 2013; Eptaimeros, Koutsoumaris, & Tsamasphyros, 2016; Khodabakhshia & Reddy, 2015; Pisano & Fuschi, 2003; Polizzotto, 2001; Wang, Zhu, & Dai, 2016; Zhu & Dai, 2012; Zhu, Wang, & Dai, 2017), and is given by:

$$\boldsymbol{\sigma}(\mathbf{x}) = \xi_1 \mathbf{C} : \boldsymbol{\varepsilon}(\mathbf{x}) + \xi_2 \int_V \alpha\left(\mathbf{x}, \mathbf{x}', \kappa\right) \mathbf{C} : \boldsymbol{\varepsilon}\left(\mathbf{x}'\right) dV' \tag{1}$$

$\boldsymbol{\sigma}$ being the nonlocal Cauchy stress tensor, and $\boldsymbol{\varepsilon}$ the infinitesimal strain tensor at a reference point $\mathbf{x}$; $\alpha(\mathbf{x}, \mathbf{x}', \kappa)$ is the kernel that represents the nonlocal behaviour which is dependent on a internal characteristic length, κ, linked to some material properties (lattice parameter, size of the grain, granular distance), $\mathbf{C}$ is the fourth-order elasticity tensor, and V is the solid domain.

The parameters ξ_1 and ξ_2 represent the volume fraction of material complying with local and nonlocal integral elasticity, respectively. Thus, the relation $\xi_1 + \xi_2 = 1$ holds. The case $\xi_2 = 0$ corresponds to the pure local elasticity approach, while $\xi_1 = 0$ deals with the fully nonlocal integral elasticity formulation.

For this last case, $\xi_1 = 0$, the nonlocal constitutive theory, introduced by Eringen (1983), is recovered. Eringen showed that, for a specific class of kernel functions, the nonlocal integral constitutive equation can be transformed into a differential form. Peddieson, Buchanan, and McNitt (2003) applied for the first time the differential Eringen nonlocal model to the Bernoulli–Euler beams, and from that, and due to its simplicity, this approach has been widely used to analyze the static, buckling, and dynamic behavior of nanostructures. It is not feasible to report here the whole of the related papers. Therefore we refer the recent reviews by Eltaher, Khater, and Emam (2016) and Rafii-Tabar, Ghavanloo, and Fazelzadeh (2016) on the application of nonlocal continuum theories to nanostructures.

However, Benvenuti and Simone (2013) found incoherent results related to the static behavior of a bar subjected to axial loads. Moreover, several authors have pointed out the paradoxical results obtained from the Eringen differential model regarding a cantilever beam when compared to other boundary conditions, both for the static (Challamel & Wang, 2008; Challamel et al., 2014; Peddieson et al., 2003; Wang, Kitipornchai, Lim, & Eisenberger, 2008; Wang & Liew, 2007) and vibrational behaviour (Lu, Lee, Lu, & Zhang, 2006).

Although several attempts have been made to overcome these paradoxical results (Challamel & Wang, 2008; Fernández-Sáez, Zaera, Loya, & Reddy, 2016), a clear picture of the problem has been pointed out by Romano, Barretta, Diaco, and de Sciarra (2017) who shown that, in the majorities of the cases, the integral formulation of the fully nonlocal elasticity theory leads to problems that have to be considered as ill-posed. These problems have no solution in general. Only when certain constitutive boundary conditions are fulfilled (Polyanin & Manzhirov, 2008; Romano et al., 2017), the integral formulation is equivalent to the differential one, and the problem has a unique solution. Therefore, Fernández-Sáez et al. (2016), Tuna and Kirca (2016a,b) and Eptaimeros et al. (2016) proposed improper solutions (numerical or analytical) for a problem which is in fact unsolvable in most of the practical cases, since the constitutive boundary conditions are not fulfilled. However, using the mixed constitutive model with $\xi_1 > 0$, the ill-posedness of the purely nonlocal problem is eliminated, and therefore true solutions can be achieved using this formulation (Romano et al., 2017).

Variational principles governing the integral form of the two-phase nonlocal approach were derived by Polizzotto (2001), and the model was applied latter to analyse several problems related to the static behaviour of nanostructures. Zhu and Dai (2012), Pisano and Fuschi (2003), and Benvenuti and Simone (2013) solved the problem of a bar subjected to static axial loads. Pisano, Sofi, and Fuschi (2009) used this integro-differential nonlocal model to derive a finite element formulation for 2D problems. More recently, with the same constitutive model, the static bending of Bernoulli–Euler beams subjected to different boundary and load conditions has been studied, via a finite element approach (Khodabakhshia & Reddy, 2015), or with an analytical model (Wang et al., 2016). The buckling of Bernoulli–Euler beams was also addressed using this constitutive formulation (Zhu et al., 2017).

The vibrational behaviour of Euler–Bernoulli beams involving the two-phase Eringen nonlocality has been studied by Eptaimeros et al. (2016), using a *FEM* approach to obtain the eigenfrequencies for different boundary conditions. They analyzed the effects of different nonlocal parameters in the dynamic response of the beams.

In this paper we formulate and analytically solve the problem of the free in-plane (axial and bending) vibrations of a beam using the mixed local/nonlocal Eringen elasticity theory. The movement equations have been obtained using the Hamilton's Principle, leading to two uncoupled integro-differential eigenvalue problems corresponding to axial and bending vibrations, respectively. For the case of axial vibrations the integro-differential eigenvalue problem was transformed to a fourth-order differential equation with four boundary conditions: two of them correspond to the classical ones (one for