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Bifurcation instabilities in finite bending of circular cylindrical shells

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ABSTRACT

This work re-visits the finite pure bending problem for circular cylindrical shells within the elastic range. The interest here is primarily directed towards the bifurcation instabilities of such configurations when the progressive flattening of the cylindrical cross-section is explicitly taken into account (the so-called *Brazier effect*). By coupling Reissner's axisymmetric solution to the buckling equations for a quasi-shallow toroidal shell we formulate a novel boundary-value problem able to capture such bifurcations. Numerical simulations of this problem confirm that buckling occurs before the usual limit-point instability is reached, while singular perturbation methods allow us to obtain simple asymptotic approximations for the critical curvature and bending moment associated with the bifurcations.

1. Introduction

Problems most commonly encountered in the theory of elasticity can be broadly divided in two groups. A first such category includes *deformation problems*, in which the loads and boundary conditions are prescribed and one is interested in calculating the corresponding mechanical and kinematical fields. However, there are many situations that do not fit neatly into this mould like, for instance, those lacking uniqueness or admitting multiple solutions. Thus, a second large group consists of *bifurcation problems*; buckling, cavitation and, in general, all instabilities fall into this latter class.

Historically, there has been a poor distinction between these two rather different classes of problems. While it may be argued that such a distinction is artificial, it is very seldom in the literature that a calculated deformation is tested for stability against either general or particular perturbations; strictly speaking, such a check would be desirable in order to ascertain the range of validity for the solutions obtained.

The author and his associates have carried out a fairly extensive study into the bifurcation properties of various Lamé problems within the plane-stress framework of linear elasticity (Coman & Haughton, 2006; Coman, 2007; 2009; Coman & Bassom, 2008). More recently those results have been extended to nonlinear axisymmetric solutions for thin plates and shells (Coman & Bassom, 2016a; 2016b; 2017; Coman, Matthews, & Bassom, 2015). Despite the fact that more often than not the associated boundary-value problems are intractable by analytical means, experience has showed that invariably the asymptotic limits of the corresponding bifurcation equations are still amenable to a whole range of singular perturbation techniques (e.g., boundary layers, multiple scales, and WKB methods). In the present study we are motivated to re-visit the classical problem of pure bending for circular cylindrical shells in light of the aforementioned comments. In particular,

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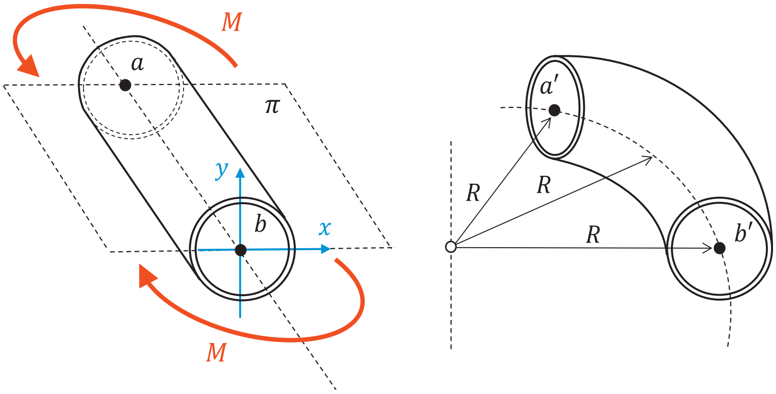

Fig. 1. Pure bending of a circular cylindrical shell.

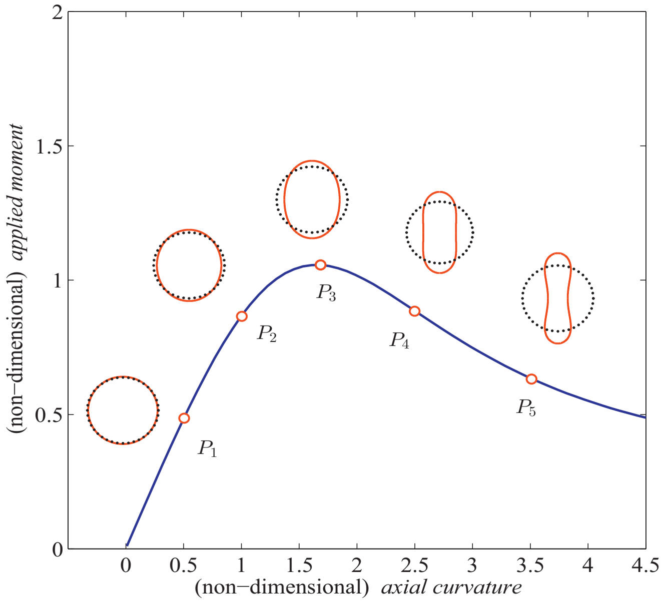

Fig. 2. Typical behaviour of the applied bending moment vs. axial curvature for relatively long circular cylinders subjected to pure bending. The round markers labelled P_j ($j = 1, 2, \dots, 5$) correspond to an arbitrary set of points on the moment-curvature graph, while the superimposed closed curves show the deformation of the cross-section associated with those points. To highlight the contrast between these different cases, the undeformed circular configuration is also showed as a dotted curve.

we consider Reissner's nonlinear deformation solution for this problem (Reissner, 1961; Reissner & Weinitschke, 1963) and explore the existence of possible bifurcations.

The scenario considered in the next sections appears sketched in Fig. 1: an initially straight and long circular cylindrical shell (left) is subjected to terminal systems of forces that are each statically equivalent to a moment M acting in the plane of curvature of the shell (labelled '$\boldsymbol{\pi}$' in the figure). As a result of this loading, the axis of the tube is bent into an arc of a circle of radius R (i.e., the line ab is mapped to $a'b'$) – see the sketch on the right in Fig. 1. This configuration was first studied by Brazier (1927), who showed that the cross-sections flatten progressively as R is decreased, an effect which is intimately linked to the longitudinal tension and compression resisting the applied bending moment; this phenomenon is usually referred to as the *Brazier effect*.

The most salient feature of the solution given by Brazier (1927) is the nonlinear softening relationship between the applied bending moment and the average axial curvature of the bent tube – see Fig. 2. The presence of the limit point on this curve could suggest that the structure will collapse once the bending moment (or the curvature) reaches the threshold associated with that particular location. However, this is not always the case. As already demonstrated by the experimental data provided by Brazier (see Fig. 4 in his work, as well as the comments on page 110, penultimate paragraph), the compressed side of the cylinder tends to experience a buckling-type bifurcation that results in sudden failure, just before the limit point is reached. Many studies related to various extensions of Brazier's effect have ignored this aspect and have focused exclusively on deriving moment-curvature relations for configurations that included additional effects: internal pressure