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Nonlinear mechanics of doubly curved shallow microshells

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ABSTRACT

This paper performs a thorough investigation on the nonlinear size-dependent bending characteristics and natural frequencies of doubly curved shallow microshells. A nonlinear continuous model for a general doubly curved microshell is developed on the basis of Donnell's nonlinear shell theory and in the framework of the modified couple-stress strain gradient theory. In particular, the doubly curved microshell equations of motion of partial differential type are derived while accounting for geometric nonlinearities and small-scale effects. The continuous model is transformed into a discretised set of equations via application of the two-dimensional Galerkin technique. A large number of modes are retained in both linear and nonlinear investigations of microshells to ensure converged and reliable results. The linear natural frequencies are reported for various microshells of rectangular and square bases. The nonlinear static deflection curves for both in-plane and out-of-plane displacements are constructed and the effects of different parameters, such as the radius of curvature, sign of the radius of the curvature, and the length-scale parameter are examined.

1. Introduction

The prevalent applications of microelectromechanical systems (MEMS), for instance in microresonators, microactuators, microswitches, microsensors, microgyroscopes, and micromirrors, have motivated a substantial amount of research on this topic. MEMS devices consist of integrated electrical and mechanical components, the latter being usually microbeams, microplates, or microshells (Akgöz & Civalek, 2011, 2013; Asghari, Kahrobaiyan, & Ahmadian, 2010; Baghani, 2012; Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani, BehboodiJouybari, & Dehrouyeh, 2016; Farokhi, Ghayesh, & Amabili, 2013a; Farokhi, Ghayesh, & Gholipour, 2017; Farokhi, Ghayesh, Gholipour, & Hussain, 2017; Ghayesh & Farokhi, 2015a, 2017; Ghayesh, Amabili, & Farokhi, 2013a; Ghayesh, Farokhi, & Amabili, 2013a; Ghayesh, Farokhi, Gholipour, Hussain, & Arjomandi, 2017; Hosseini & Bahaadini, 2016; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008; Mojahedi & Rahaeifard, 2016; Shafiei, Kazemi, & Ghadiri, 2016; Taati, 2016; Zhang & Meng, 2007). Hence, proper understanding the behaviour of these micromechanical components is essential toward development of reliable models for different types of MEMS. It has been developed theoretically and verified experimentally that microstructures show size-dependent behaviour due to their micro-scale size (Fleck, Muller, Ashby, & Hutchinson, 1994; Haque & Saif, 2003; McFarland & Colton, 2005). The higher-order theories of continuum mechanics are capable of capturing this

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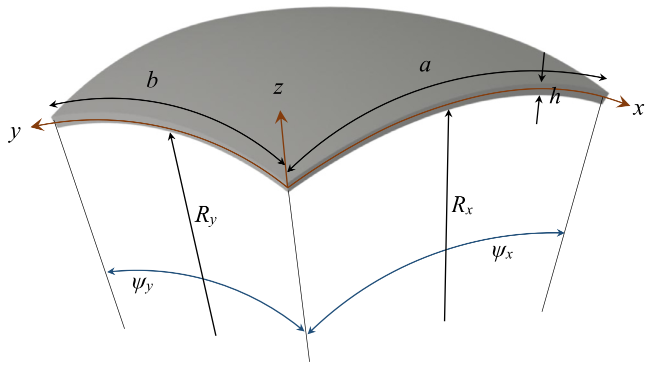

Fig. 1. Schematic representation of a doubly curved microshell in an orthogonal curvilinear coordinate.

size-dependent behaviour (Dai, Wang, & Wang, 2015; Dehrouyeh-Semnani, Dehrouyeh, Torabi-Kafshgari, & Nikkhah-Bahrami, 2015; Farokhi & Ghayesh, 2015a, 2015b; Farokhi, Ghayesh, & Amabili, 2013b; Ghayesh & Amabili, 2014; Ghayesh & Farokhi, 2015b; Ghayesh, Farokhi, & Amabili, 2013b, 2014; Gholipour, Farokhi, & Ghayesh, 2015, 2016; Li & Pan, 2015; Şimşek, 2010; Tang, Ni, Wang, Luo, & Wang, 2014); the modified couple stress theory (MCST) (Farokhi & Ghayesh, 2017a, 2017b; Ghayesh, Farokhi, & Gholipour, 2017a, 2017b; Yang, Chong, Lam, & Tong, 2002) is employed in this study.

The literature on the topic of the bending/vibration/buckling of microshells is not large. For instance, Lou, He, Wu, and Du (2016) investigated the buckling behaviour of simply-supported microshells made of functionally graded (FG) materials, subject to both axial and radial loads; they accounted for small-size effects utilising the employing MCST. Sahmani, Ansari, Gholami, and Darvizeh (2013) analysed the size-dependent dynamic stability of simply supported shear deformable cylindrical microshells made of FG materials. Tadi Beni, Mehralian, and Razavi (2015) developed the size-dependent equations of motion for an FG cylindrical shell employing MCST and studied the free vibrations of a simply supported system.

All the valuable studies mentioned above, conducted linear free vibration or buckling analysis on cylindrical shells. The present study, for the first time, examines the *nonlinear bending* and free vibration characteristics of *doubly curved* shallow microshells. Additionally, for the first time, the expressions for the higher-order symmetric rotation gradient tensor are obtained properly for a doubly curved microshell. After deriving the nonlinearly coupled equations of motion, a two-dimensional Galerkin technique is utilised to discretise the model, while retaining a large number of modes in both linear and nonlinear studies. The free vibration characteristics of various doubly curved microshells of rectangular and square bases are examined. Furthermore, the nonlinear bending response of the doubly curved microshell is analysed by constructing the bending deflection curves for both in-plane and out-of-plane displacements, and examining the effect of different parameters.

2. Model development for doubly curved microshells

The schematic of a doubly curved microshell is illustrated in Fig. 1. The microshell has a rectangular base and is described in an orthogonal curvilinear coordinate system (x, y, z), with coordinates defined as $x = R_x\psi_x$ and $y = R_y\psi_y$. R_x and R_y denote the principal radii of curvature while ψ_x and ψ_y represent the angular coordinates. The curvilinear dimensions of the microshell in the x and y directions are shown by a and b, respectively; h stands for the microshell thickness. The microshell middle surface displacements are represented by w, u, and v, in the z, x, and y directions.

In what follows, the nonlinear equations of motion for a doubly curved shallow microshell are derived making use of the MCST and Donnell's nonlinear shell theory. The displacements of a generic point of the shallow microshell in terms of the middle surface displacements, based on Donnell's theory, are given by

$$U_x = -z\partial w/\partial x + u, \quad U_y = -z\partial w/\partial y + v, \quad U_z = w. \tag{1}$$

In order to construct the potential energy of the doubly curved shallow microshell in the framework of the MCST, one needs to formulate the classical stress and strain tensors, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$, respectively, as well as the higher-order counterparts, i.e. the deviatoric part of the symmetric couple-stress tensor $\mathbf{m}$ and the symmetric rotation gradient tensor $\boldsymbol{\chi}$ (Yang, Chong, Lam, & Tong, 2002). It is important to note that, in order to be consistent with Donnell's nonlinear shell theory assumptions, the terms $(1+z/R_y)$ and $(1+z/R_x)$ are replaced by 1, in the *final* formulations for $\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$.

In what follows, the components of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$ are formulated in the curvilinear coordinate defined in Fig. 1. In all the formulations of this section, $\partial/\partial x$ and $\partial/\partial y$ are equivalent to $\partial/(R_x\partial\psi_x)$ and $\partial/(R_y\partial\psi_y)$, respectively. The nonzero components of the strain tensor $\boldsymbol{\varepsilon}$, based on Donnell's nonlinear shell theory (assuming $(1+z/R_y)\approx 1$ and $(1+z/R_x)\approx 1$), are given by

$$\varepsilon_{xx} = \left(\frac{\partial u}{\partial x} + \frac{w}{R_x}\right) + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2},$$