International Journal of Engineering Science 118 (2017) 16–27

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

# Transport of platelets induced by red blood cells based on mixture theory

Wei-Tao Wu[a], Nadine Aubry[b], Mehrdad Massoudi[c,*], James F. Antaki[a]

[a] Department of Biomedical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA
[b] Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA 02115, USA
[c] U. S. Department of Energy, National Energy Technology Laboratory (NETL), 626 Cochrans Mill Road, P. O. Box 10940, Pittsburgh, PA 15236, USA

ARTICLE INFO

*Article history:*
Received 24 October 2016
Revised 25 April 2017
Accepted 4 May 2017

*Keywords:*
Platelets
Blood flow
RBCs
Micro-channel
Mixture theory
Two-fluids
Thrombosis

ABSTRACT

The near-wall enrichment of platelets strongly influences thrombus formation *in vivo* and *in vitro*. This paper develops a multi-constituent continuum approach to study this phenomenon. A mixture theory model is used to describe the motion of plasma and red blood cells (RBCs) and the interactions between the two components. A transport model is developed to study the influence of the RBCs field on the platelets. The model is used to study blood flow in a rectangular micro-channel, a sudden expansion micro-channel, and a channel containing micro crevices (representing a practical problem encountered in most blood-wetted devices). The simulations show that in the rectangular channel the concentration of the platelets near the walls is about five times higher than the concentration near the channel centerline. It is also noticed that in the channel with crevices, a large number of platelets accumulate in the deep part of the crevices and this may serve as nidus for excessive thrombus formation occurring in medical devices.

Published by Elsevier Ltd.

## 1. Introduction

A thrombus, sometimes called a blood clot, is a natural response of the body to an injury by attempting to prevent bleeding; however, excessive thrombosis is responsible for device malfunction and diseases such as stroke. Thrombosis in the coronary artery can lead to heart attacks; a thrombus can also be transported to the brain by blood circulation causing cardiogenic strokes (Furie & Furie, 2008; Handin, 2005). Thrombus generation in blood-related medical devices can reduce the efficiency of the devices and this may lead to their malfunction (Jaffer, Fredenburgh, Hirsh, & Weitz, 2015; Kirklin et al., 2012; Reviakine et al., 2016; Slaughter et al., 2009).

Thrombosis is usually initiated as platelet adhesion on biological or artificial surfaces. Thrombus initiation and growth in vessels or on medical devices is strongly influenced by the number density of the platelets near the walls and surfaces (Cito, Mazzeo, & Badimon, 2013; Skorczewski, Erickson, & Fogelson, 2013; Weller, 2008; Yang, Jäger, Neuss-Radu, & Richter, 2016). In small blood vessels, the platelets tend to move away from the center of the blood vessels and accumulate near the walls. Such a transverse migration of platelets is believed to be closely related to the motion of the red blood cells (RBCs), where opposite to the movement of the platelets towards the walls, the RBCs tend to move away from the walls of the vessels (AlMomani, Udaykumar, Marshall, & Chandran, 2008; Goldsmith & Turitto, 1986; Turitto, Benis, & Leonard, 1972).

* Corresponding author.
*E-mail address:* mehrdad.massoudi@netl.doe.gov (M. Massoudi).

http://dx.doi.org/10.1016/j.ijengsci.2017.05.002
0020-7225/Published by Elsevier Ltd.

Through many mesoscale simulations, the non-uniform distribution of the platelets is believed to be mainly attributed to the collisions between the RBCs and the platelets (AlMomani et al., 2008; Reasor, Mehrabadi, Ku, & Aidun, 2013; Skorczewski, Erickson, & Fogelson, 2013). Direct experimental measurements also indicate that when RBCs are introduced in the flow, the platelets migration to the vessel walls increases intensively (AlMomani et al., 2008; Cadroy & Hanson, 1990; Joist, Bauman, & Sutera, 1998; Peerschke, Silver, Weksler, Yin, Bernhardt & Varon, 2007; Turitto & Weiss, 1980).

In order to describe or to be able to predict the non-uniform distribution of the platelets, we need to use a multi-component theory to model blood flow so that important quantities such as the velocity and the volume fraction fields of the RBCs can be studied. That is, a single-phase blood model is not able to predict the migration of the blood cells. Although many numerical studies of the near-wall platelets enrichment have been done using mesoscale simulations, in most engineering-scale applications, due to the high computational cost of the mesoscale simulations, a continuum model can be used. In the present study, using the multi-component blood model recently developed by Wu, Aubry, Massoudi, Kim, and Antaki (2014) and Wu, Yang, Antaki, Aubry, and Massoudi (2015), we introduce and advocate a transport flux equation to study and model the non-uniform distribution of the platelets .

## 2. The mathematical model

A thrombus usually indicates a blood clot attached to the (damaged) vascular walls. There are a few theoretical models of thrombus formation (see Anand & Rajagopal, 2004, 2002; Anand, Rajagopal, & Rajagopal, 2006a,b, 2008; Flamm & Diamond, 2012; Kuharsky & Fogelson, 2001; Wu et al., 2017). In general, it is believed that the RBCs distribution can influence the distribution of the platelets which in turn is closely related to the thrombus formation (Anand et al., 2008). In this paper, we assume that blood is a multi-component fluid composed of red blood cells, plasma and platelets. The presence and influence of other components such as the white bloods cells are ignored. Furthermore, we assume that the motion of the RBCs and the plasma are governed by the conservation equations based on mixture theory, whereas for the motion of the platelets we propose a transport flux (a convection-diffusion) equation. These conservation equations are used to obtain the velocity fields for the plasma and the RBCs, the hematocrit (RBCs concentration), and the pressure fields. The equations of motion used in this paper are based on the Mixture Theory (theory of interacting continua) as given in (Massoudi, Kim, & Antaki, 2012; Rajagopal & Tao, 1995; Wu, Aubry, & Massoudi, 2014). The classical theory of mixtures is described in detail in books by Rajagopal and Tao (1995), Truesdell (1984), and in review articles by Bowen (1976) and Atkin and Craine (1976a,b).

### *2.1. Governing equations*

#### *2.1.1. Conservation of mass*

In the absence of thermo-chemical and electromagnetic effects, the governing equations consist of the conservation of mass, linear momentum and angular momentum. In the Eulerian form, conservation of mass, for each component, is expressed as (Bowen, 1976):

$$\frac{\partial \rho_f}{\partial t} + div\left(\rho_f \boldsymbol{v}_f\right) = 0 \tag{1}$$

$$\frac{\partial \rho_s}{\partial t} + div(\rho_s \boldsymbol{v}_s) = 0 \tag{2}$$

where $\frac{\partial}{\partial t}$ is the derivative with respect to time, *div* is the divergence operator and $\boldsymbol{v}$ is the velocity field. The subscript ‘f’ refers to the fluid (plasma), and ‘s’ to the solid particles, representing the RBCs. The densities of the two constituents are: $\rho_f = (1-\phi)\rho_{f0}$, $\rho_s = \phi\rho_{s0}$, where $\rho_{f0}$ and $\rho_{s0}$ are the pure density of the plasma and the RBCs in the reference configuration, respectively, and $\phi$ is the volume fraction (hematocrit) of the RBCs. $\rho_{f0}$ and $\rho_{s0}$ are constant in this paper.

#### *2.1.2. Conservation of linear momentum*

The balance of the linear momentum can be written as:

$$\rho_f \frac{D\boldsymbol{v}_f}{Dt} = div\boldsymbol{T}_f + \rho_f \boldsymbol{b}_f + \boldsymbol{f}_I \tag{3}$$

$$\rho_s \frac{D\boldsymbol{v}_s}{Dt} = div\boldsymbol{T}_s + \rho_s \boldsymbol{b}_s - \boldsymbol{f}_I \tag{4}$$

where, $\frac{D}{Dt}$ is the material derivative. For any scalar $\beta$, $\frac{D\beta}{Dt} = \frac{\partial \beta}{\partial t} + \boldsymbol{v} \cdot grad\,\beta$; for any vector $\boldsymbol{w}$, $\frac{D\boldsymbol{w}}{Dt} = \frac{\partial \boldsymbol{w}}{\partial t} + (\text{grad}\boldsymbol{w})\boldsymbol{v}$, where ‘grad’ is the gradient operator, $\boldsymbol{T}_f$ and $\boldsymbol{T}_s$ are the partial Cauchy stress tensors for the plasma and the RBCs, respectively. $\boldsymbol{f}_I$ represents the interaction force (exchange of momentum) between the two components, and $\boldsymbol{b}_f$ and $\boldsymbol{b}_s$ refer to body forces. The balance of the angular momentum implies that, in the absence of couple stresses, the total Cauchy stress tensor is symmetric. To close these equations, constitutive relations are needed for the stress tensors $\boldsymbol{T}_f$ and $\boldsymbol{T}_s$ and the interaction force $\boldsymbol{f}_I$.