International Journal of Engineering Science 118 (2017) 70–81

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Effective conductivity of materials with continuous curved fibers

Dmytro Kuksenko, Borys Drach*

Department of Mechanical and Aerospace Engineering, New Mexico State University, Las Cruces, NM USA

ARTICLE INFO

Article history:
Received 21 May 2017
Accepted 2 June 2017

Keywords:
Micromechanics
Conductivity contribution tensor
Curved fiber
Effective properties
Finite element analysis

ABSTRACT

Overall conductivity of composites reinforced with in-phase continuous sinusoidal fibers of circular cross-sections is investigated. The effects of the fiber/matrix conductivity contrast, crimp ratio and relative fiber radius are studied. Two approaches are employed for the analysis: direct finite element analysis based on curvilinear periodic unit cells, and micromechanical homogenization based on the conductivity contribution tensor combined with the non-interaction and Mori-Tanaka schemes. In addition, an approach to approximation of the conductivity contribution tensor of a single fiber using an equivalent set of ellipsoids is presented. Comparison of the direct numerical analysis results with micromechanical homogenization shows that the latter approach can be used to estimate all components of the effective conductivity tensor of the considered composite material system with good accuracy. However, the results also indicate that the choice of the micromechanical homogenization scheme for this type of reinforcement depends on the conductivity component being approximated and the fiber/matrix conductivity contrast.

1. Introduction

Structures with continuous curved fibers are found in textile composites (Drach, Drach, & Tsukrov, 2014; Rinaldi, Blacklock, Bale, Begley, & Cox, 2012), natural and engineered biological materials (Abolfathi, Naik, Sotudeh Chafi, Karami, & Ziejewski 2009; Caves et al, 2010) and stretchable electronics (Kim, Ghaffari, Lu, & Rogers, 2012; Rogers, Someya, & Huang, 2010). While the mechanical response of such structures has been studied extensively (see, for example, Chan & Wang, 1994; Garnich & Karami, 2004; Stig & Hallström, 2013; Tsai, Zhang, Jack, Liang, & Wang, 2011), research on conductivity is limited.

Published approaches on modeling of the overall conductivity of composites with continuous wavy reinforcement can be grouped into two categories: homogenization based on the thermal-electrical analogy and homogenization via finite element analysis (FEA). It is worth mentioning that most of such works focus on textile and braided composites in which the reinforcement is represented by bundles of thousands of fibers, not individual fibers, which are discussed in this paper.

The idea of the thermal-electrical analogy method is based on the similarity between the equations governing temperature and electric potential distributions. As a result, the overall thermal conductivity is estimated from the analysis of the equivalent electrical resistance using Ohm's law. The reported approaches vary with respect to the level of detail of reinforcement architecture representation. Dasgupta and Agarwal (1992) and Dasgupta, Agarwal, and Bhandarkar (1996) focused on the effective orthogonal conductivity of a plain-weave fabric composite. The authors used optical micrographs to develop

* Corresponding author.
E-mail addresses: kuksenko@nmsu.edu (D. Kuksenko), borys@nmsu.edu (B. Drach).

http://dx.doi.org/10.1016/j.ijengsci.2017.06.001
0020-7225/

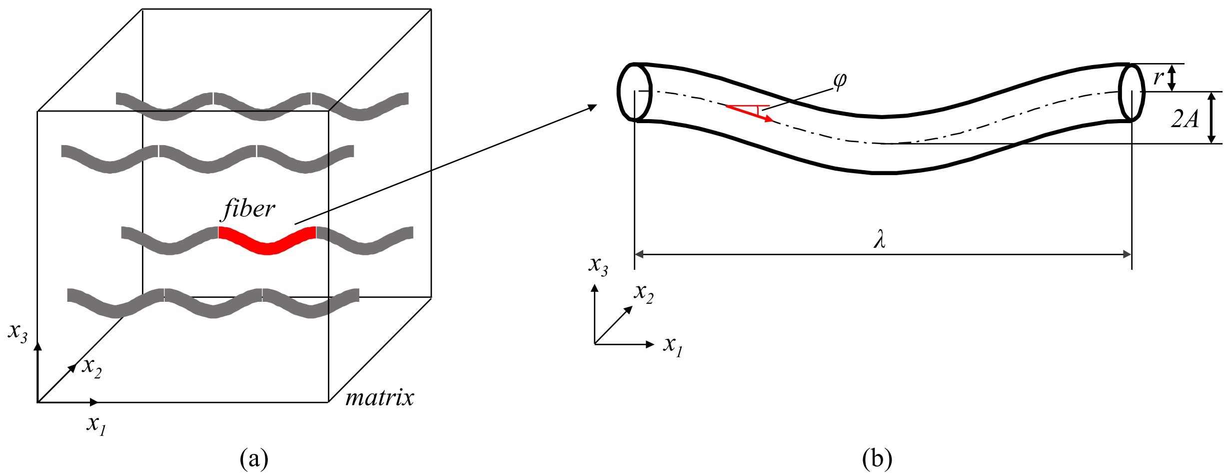

Fig. 1. Geometry of the problem under consideration: (a) configuration of the composite reinforced with continuous sinusoidal fibers; (b) geometry of an individual fiber: A is the amplitude of the fiber's path, r is the radius, λ is the wavelength, and φ is the angle between the local tangent of the fiber's path and x_1 direction.

geometric representations of the composite. In the approach, the overall conductivity is calculated by numerically integrating the thermal resistances of thin differential elements. The effective conductivities of the latter were estimated from the equivalent series-parallel "thermal resistance network" . The analytical predictions were compared with experimental and numerical results, and a good correlation was observed. Ning and Chou (1995) derived closed-form expressions for the in-plane effective thermal conductivities of plain-weave fabric composites using the thermal-electrical analogy and a simplified piecewise linear approximation of the reinforcement geometry. The authors expanded their analysis to other reinforcement geometries including a twill weave and several configurations of satin weave fabrics in a later publication, see Ning and Chou (1998). A similar approach based on a simplified tow path geometry was used for prediction of effective thermal conductivities of fabrics and fabric composites by Yamashita, Yamada, and Miyake (2008).

Numerical homogenization to determine the effective thermal conductivity via FEA is presented in Woo and Goo (2004) for satin weave carbon/phenolic composites. The authors utilized a simplified representation of the composite reinforcement's three-dimensional (3D) geometry. Tomkova, Sejnoha, Novak, and Zeman (2008) used FEA to determine the overall conductivity of porous plain weave carbon/carbon composites. Microscopy data was employed by the authors to generate realistic two-dimensional (2D) periodic unit cells representing cross-sections of the composite. Jiang, Xu, Cheng, Lu, and Zeng (2014) present an implementation of a regular-grid FEA approach for predicting thermal conductivity and temperature distribution in 3D braided composites. The simulations results are compared with experimental measurements and a good correspondence is observed. Gou, Dai, Li, and Tao (2015) performed a parametric study using FEA to determine the effect of matrix porosity and fiber volume fraction on the overall conductivity of the plain weave carbon/silicon carbide composites. Dong, Liu, Pan, Gu, and Sun (2016) investigated thermal conductivity of 2.5D angle-interlock woven composites numerically and experimentally.

In this paper, we study the overall conductivity (thermal or electrical) of a composite with continuous sinusoidal fibers embedded in the matrix material, see Fig. 1a. Both matrix and fiber are assumed to have isotropic material properties. The geometry of an individual fiber (Fig. 1b) is described by two parameters: crimp ratio CR and relative radius $\tilde{r}$. The former is defined as the ratio of the amplitude A to the wavelength λ of the sinusoidal fiber path:

$$CR = \frac{A}{\lambda}; \tag{1.1}$$

the latter is introduced as the ratio of the fiber's radius r to its wavelength λ:

$$\tilde{r} = \frac{r}{\lambda}. \tag{1.2}$$

The paper is organized as follows. In Section 2, we use the direct finite element analysis to analyze the effects of micromechanical parameters of composites with sinusoidal fibers on the overall thermal conductivity. The concept of the conductivity contribution tensor is introduced in Section 3. In the same section, we present the numerically calculated tensors of individual sinusoidal fibers and propose an analytical approximation procedure to estimate the tensor components without FEA. We compare the overall conductivities of composites with in-phase sinusoidal fibers calculated using the proposed approximation with the numerical results in Section 4. The conclusions of the research are presented in Section 5.