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Numerical simulations of classical problems in two-dimensional (non) linear second gradient elasticity

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A two-dimensional solid consisting of a linear elastic isotropic material is considered in this paper. The strain energy is expressed as a function of the strain and of the gradient of strain. The balance equations and the boundary conditions have been derived and numerically simulated for those classical problems for which an analytical solution is available in the literature. Numerical simulations have been developed with a commercial code and a perfect overlap between the results and the analytical solution has been found. The role of external edge double forces and external wedge forces has also been analyzed. We investigate a mesh-size independency of second gradient numerical solutions with respect to the classical first gradient one. The necessity of a second gradient modelling is finally shown. Thus, we analyze a non-linear anisotropic problem, for which experimental evidence of internal boundary layer is shown and we prove that this can be related to the second gradient modelling.

1. Introduction

The introduction of higher order gradients of the strain into the constitutive law for the internal energy leads to a partial differential equation of higher orders and the Galerkin method requires a higher regularity of the interpolation scheme, see e.g. Bilotta, Formica, and Turco (2010). The reason of introducing higher-order gradient theories is based on different points of view, see e.g. Aminpour and Rizzi (2015, 2016a,b); Aminpour, Rizzi, and Salerno (2014). The first example is referred to the case related to strongly localized deformation features (Altenbach, Eremeyev, and Morozov, 2010; Cuomo, Contrafatto, and Greco, 2014; Luca, 2015; Luca, 2016; Roveri, Carcaterra, & Akay, 2009) and references therein. In such cases, it is reasonable to complement the displacement field with some additional kinematical descriptors (Gabriele, Rizzi, & Varano, 2012, 2016, 2014; Piccardo, Ranzi, & Luongo, 2014a,b; Pignataro, Ruta, Rizzi, & Varano, 2010; Rizzi & Varano, 2011), which leads to the so-called micromorphic models, see also Ionel-Dumitrel, Patrizio, Angela, Placidi, and Giuseppe (2015); Madeo, Neff, Ghiba, Placidi, and Rosi (2015a,b); Neff, Ghiba, Madeo, Placidi, and Rosi (2014). Another possibility is to consider higher order gradient theories, in which the deformation energy depends on second and/or higher gradients of the

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displacement (dell'Isola, Madeo, & Placidi, 2012; Ferretti, Madeo, dell'Isola, & Boisse, 2014; Madeo, dell'Isola, & Darve, 2013). This is done in the literature for both monophasic systems (see Auffray, dell'Isola, Eremeyev, Madeo, and Rosi, 2015; Rosi, Giorgio, and Eremeyev, 2013, in which continuous systems are investigated, and Alibert and Corte (2015); Seppecher, Alibert, and dell'Isola (2011) for cases of lattice/woven structures) and for bi-phasic (see e.g. Andreaus, Giorgio, & Lekszycki, 2014; dell'Isola, Guarascio, & Hutter, 2000; Misra, Parthasarathy, Singh, & Spencer, 2015; Sciarra, dell'Isola, & Coussy, 2007) or granular materials (Misra & Singh, 2014; Yang & Misra, 2012). The second example is referred to the fact that, unlike classical Cauchy continua, second and higher order continua can respond to concentrated forces and generalized contact actions (see e.g. in Carcaterra & Roveri, 2012; de Oliveira Góes, de Castro, & Martha, 2014). It is worth to be noted that it is also possible to conceive a framework in classical elasticity (see, e.g., Bulíček, Málek, Rajagopal, and Walton, 2015; Mindlin, 1936 and references therein) in which concentrated forces are possible. However, with a greater theoretical and numerical efforts. The third example is becoming increasingly important for practical and applicative reasons in the last years, as the novelties in manufacturing procedures (due to, e.g., 3D printing, self assembly etc.) are making possible the realization of a much wider class of new architectured materials (Del Vescovo & Giorgio, 2014; Milton & Seppecher, 2012). In these cases, the deficiencies of classical approaches when the material behaviour exhibits size-scale effects is investigated in Sansour and Skatulla (2009); Scerrato, Giorgio, and Rizzi (2016); Scerrato, Zhurba Eremeeva, Lekszycki, and Rizzi (2016), and in Neff, Jeong, and Ramézani (2009) a novel invariance requirement (micro-randomness) in addition to isotropy is formulated, which implies conformal invariance of the curvature. In general, new theories are put into place when existing theories prove to be inadequate to describe some observed phenomenon. Such new theories however have to lead to well posed problems in the sense that the governing equations and boundary conditions lead to solvable problems. The papers (Alibert & Corte, 2015; Mareno & Healey, 2006; Pideri & Seppecher, 1997) already proved that the problems we study here is indeed well-posed.

A survey of variational principles, which form the basis for computational methods in both continuum mechanics and multi-rigid body dynamics is presented in Atluri and Cazzani (1995) and numerical investigation of structures of the type considered also requires special attention and the development of novel techniques (Assante & Cesarano, 2014; Bilotta & Turco, 2009; Cesarano, 2014; Cesarano & Assante, 2014; Cesarano, Fornaro, & Vazquez, 2016; Garusi, Tralli, & Cazzani, 2004; Greco & Cuomo, 2015) or the proper employment of the existing ones (see for instance Terravecchia, Panzeca, and Polizzotto, 2014, where Galerkin Boundary Element Method is used to address a class of strain gradient elastic materials). The objective of the contribution (Javili, Dell'Isola, & Steinmann, 2013) is to formulate a geometrically nonlinear theory of higher-gradient elasticity accounting for boundary (surface and curve) energies. To reduce the computational costs and avoid the macroscopic grid sensitivity, an adaptive multiscale technique is developed for strain localization analysis of periodic lattice truss materials in Zhang, Wu, and Zheng (2012). In Zervos (2008) a general finite element discretization of micromorphic Mindlin' s elasticity is presented. The behavior of all elements is also examined at the limiting case of strain gradient elasticity. The numerical solution of second gradient elasticity equations with a displacement-based finite element method requires the use of C1-continuous elements, that motivates the implementation of the concept of isogeometric analysis in Fischer, Klassen, Mergheim, Steinmann, and Müller (2011). In Papanicolopulos, Zervos, and Vardoulakis (2009) a new C1 hexahedral element which is the first three-dimensional C1 element ever constructed and give excellent rates of convergence in a benchmark (without edge forces) boundary value problem of gradient elasticity. In Papanicolopulos and Zervos (2012) a methodology by which C1 elements, such as the TUBA 3 element proposed by Argyris, Fried, and Scharpf (1968), can be constructed is presented. This kind of elements are largely present in the literature of strain gradient elasticity (Akarapu & Zbib, 2006; Dasgupta & Sengupta, 1990; Fischer, Mergheim, & Steinmann, 2010; Zervos, Papanastasiou, & Cook, 1998; Zervos, Papanastasiou, & Vardoulakis, 2001; Zervos, Papanicolopulos, & Vardoulakis, 2009).

From a general point of view a comparison between analytical and numerical solution is needed to check the quality of the used code. In other words, it means that the code has good performances and there is a degree of reliability to be assigned to it. In this paper a two-dimensional solid consisting of a linear elastic isotropic material is considered. The strain energy is expressed as a function of the strain and of the gradient of strain. The aim of the paper is to present the possibility to numerically simulate general strain gradient elasticity by the use of a commercial code that includes the Argyris shape functions. This is done with the use of benchmark boundary value problem for which an analytical solution exist. We remark that in such a 2-dimensional benchmark boundary value problem wedge forces are present. We remark a perfect overlap between the numerical results and the analytical solution of the benchmark classical boundary value problem. The role of external edge double forces and external wedge forces has also been analyzed. We also investigate a mesh-size independency of second gradient numerical solutions with respect to the classical first gradient one.Finally, we show an experimental evidence of the necessity of second gradient modeling. We show the experimental results (dell'Isola, Giorgio, Pawlikowski, & Rizzi, 2016) of a bias test on a pantographic structure. In particular, we show that the boundary layer experimentally observed can be numerically achieved by a non-null second gradient constitutive coefficient and the largeness of such a boundary layer can be used to identify such a second gradient parameter. For a better representation of the state of the art of material identification of second gradient coefficients the reader is invited to see Auffray, Bouchet, and Brechet (2009); Luca, Andreaus, Corte, and Lekszycki (2015); Placidi, Andreaus, and Giorgio (2016).