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An explicit macro-micro phase-averaged stress correlation for particle-enhanced composite materials in loaded structures

T.I. Zohdi

Department of Mechanical Engineering, University of California, Berkeley, CA, 94720-1740, USA

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ABSTRACT

In many structural applications, there is growing industrial interest in using new microscale particle-enhanced composite materials. During the design development of such machinery and materials it is advantageous to ascertain the stress load-sharing between the added particles and the binding matrix, in order to make estimates of the device's useful life and the material performance. Accordingly, in this work, we correlate the phase-averaged microstructural stress levels caried by the particles and matrix to the macrostructural loading. Model problems are studied whereby macroscale stresses are determined using a structural-scale model and the microscale stresses are then computed by constructing stress concentration functions. This provides analysts with a novel and easy to use design framework that clearly identifies the stress contributions from the microscale and the macroscale, in order to reduce product development time and costs. Examples are provided to illustrate the approach.

1. Introduction

The industrial use of particle-enhanced composite materials in structural applications is increasing (Fig. 1). Analysts are now afforded with many particle-matrix choices for possible material combinations. However, due to the nature of such applications, experiments to determine the appropriate combinations of particle and matrix materials are time-consuming and expensive, and it is advantageous to characterize such materials analytically and computationally, in order to reduce product development time and costs.

In order to characterize the effective macroscale (structural) material response of such materials, a relation between averages,

$$\langle \boldsymbol{\sigma} \rangle_{\Omega} = \mathbb{E}^{*} : \langle \boldsymbol{\epsilon} \rangle_{\Omega}, \tag{1}$$

is sought, where

$$\langle \cdot \rangle_{\Omega} \stackrel{\text{def}}{=} \frac{1}{|\Omega|} \int_{\Omega} \cdot \, d\Omega \,, \tag{2}$$

and where, throughout the structure, the mechanical properties of microheterogeneous materials are characterized by a spatially variable elasticity tensor $\mathbb{E} = \mathbb{E}(\boldsymbol{x})$ and $\boldsymbol{\sigma}$ and $\boldsymbol{\epsilon}$ are the stress and strain tensor fields within a Representative Volume Element (RVE) of volume $|\Omega|$. The quantity $\mathbb{E}^{*}$ is known as the effective property. It is the elasticity tensor used

E-mail address: zohdi@me.berkeley.edu

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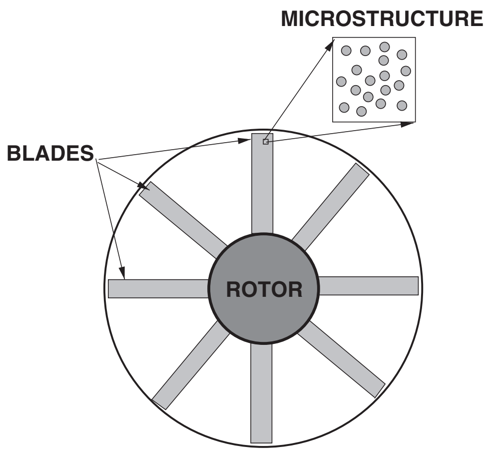

Fig. 1. An example: a "mock" rotating structure comprised of a matrix binder and particulate additives.

in usual structural (macroscale) analyses. The internal fields, which are to be volumetrically averaged, can be computed by solving a series of boundary value problems with test loadings over an RVE using the Finite Element Method. However, this is extremely computationally intensive (Zohdi & Wriggers, 2008) and oftentimes, faster approximate methods are sought. *Such approximations are important as a design tool, which is the objective of this paper.* Explicitly, in this work, we correlate the phase-averaged microstructural stress levels caried by the particles and matrix to the macrostructural loading. Model problems are studied whereby macroscale stresses are determined using a structural-scale model and the microscale stresses are then computed by constructing stress concentration functions. The objective is to provide analysts with an easy to use design framework that clearly identifies the stress contributions from the microscale and the macroscale, in order to reduce product development time and costs.

Remark 1. We will concentrate on isotropic materials in this paper. In the case of isotropic overall responses, we may write

$$\left\langle \frac{tr\boldsymbol{\sigma}}{3} \right\rangle_{\Omega} = 3\kappa^{*} \left\langle \frac{tr\boldsymbol{\epsilon}}{3} \right\rangle_{\Omega} \tag{3}$$

and

$$\langle \boldsymbol{\sigma}' \rangle_{\Omega} = 2\mu^{*} \langle \boldsymbol{\epsilon}' \rangle_{\Omega}, \tag{4}$$

where κ^* and μ^* are the effective bulk and shear moduli, $\frac{tr\boldsymbol{\sigma}}{3}$ is the dilatational stress, $\frac{tr\boldsymbol{\epsilon}}{3}$ is the dilatational strain, $\boldsymbol{\sigma}'$ is the deviatoric stress and $\boldsymbol{\epsilon}'$ is the deviatoric strain.

Remark 2. For an authoritative review of the general theory of random heterogeneous media see, for example, see Torquato (2002) for general interdisciplinary discussions, Jikov, Kozlov, and Olenik (1994) for more mathematical aspects, Hashin (1983), Mura (1993) or Markov (2000) for solid-mechanics inclined accounts of the subject, for analyses of defect-laden, porous and cracked media, see Kachanov (1993), Kachanov and Sevostianov (2005), Kachanov, Tsukrov, and Shafiro (1994), Sevostianov, Gorbatikh, and Kachanov (2001), Sevostianov and Kachanov (2008) and for computational aspects see Ghosh (2011), Ghosh and Dimiduk (2011) and Zohdi and Wriggers (2008).

Remark 3. There exist several new techniques manufacturing composite materials for deployment in complex structural shapes, such as sacrificial patterning (Singh & Singh, 2015), reinforcement with carbon nanotubes (Isaza, Sierra, & Meza, 2015), multiphase extrusion (Khalifa, Foydl, Pietzka, & Jager, 2015), microcutting arrays (Pacella et al., 2015), interlaminar glass reinforcement (Bian, Satoh, & Yao, 2015), electric field assistance (Decker & Gan, 2015) and selective laser sintering (Gu, Chang, & Dai, 2015), where the upcoming approach developed in this work could be useful in the development and optimization of these processes.

2. Effective property estimates

The literature on methods to estimate the overall macroscopic properties of heterogeneous materials dates back at least to the 1800s by the pioneering works of Maxwell (1867, 1873) and Rayleigh (1892), with an extremely important contribution being the Hashin–Shtrikman bounds during the 1960s (Hashin, 1983; Hashin & Shtrikman, 1962, 1963). The Hashin–Shtrikman bounds are the tightest possible bounds on isotropic effective responses, generated from isotropic microstructures, where the volumetric data and phase contrasts of the constituents are the only data known. They are as follows, for the bulk