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Torsion of a functionally graded material

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ABSTRACT

Within the framework of the nonlocal strain gradient theory, a size-dependent shaft model, which can account for the through-radius power-law variation of two-constituent functionally graded (FG) materials, is derived to investigate the small-scaled effects on the static and dynamic torsion behaviors. The equations of torsional motion and corresponding boundary conditions of the size-dependent FG shaft are derived in terms of the Hamilton's principle. The shaft models can account for the small-scaled effects of the inter-atomic long-range force and the microstructure deformation mechanism by introducing material length scale and nonlocal parameters. An analysis on the harmonic propagation with time torsional waves in a nonlocal strain gradient FG shaft is carried out. In the case of clamped-clamped boundary conditions, analytical solutions are obtained for the free vibration and static torsion problems of nonlocal strain gradient FG shafts. The effects of small-scaled parameters and the through-radius power-law variation of a two-constituent FG material on wave propagation, free vibration and static torsion are investigated in details.

1. Introduction

The cylindrical shaft subjected to static torque and harmonic torsional vibrations is of fundamental and engineering interest. For example, the torsional vibration (the resonant column test) can be applied for laboratory test to measure the shear modulus of material (Polyzos et al., 2015), which is needed for most of engineering design problems. A variety approaches were developed for the manufacturing processes of micro-electro-mechanical systems (MEMS). One of these manufacturing approaches is micro-manufacturing derived from its conventional macro-scaled counterparts (including micro-EDM (electro-discharge machining), laser and ion beam machining, micro-ECM (electro-chemical machining) and micro-ultrasonic machining) (Yoon & Ehmann, 2016). As a result, for example, the geometries of the shank and taper in the micro-end mill are produced commercially with a diameter as small as 50 µm (Chae, Park, & Freiheit, 2006). In the case of structural systems at small-scale, small-scaled effects have been observed on the static and dynamic torsion behaviors. Because these micro-scaled devices involving shaft-like elements are often subjected to torsional loads, the size-dependent torsional deformation and vibration of cylindrical shafts needs to be studied and understood for potential designs and applications of micro-scaled shaft-type structures.

There are two mainly theoretical approaches (atomistic and non-classical continuum mechanics) developed for the analyses of small-scaled effect on the mechanical behaviors of smalled-scaled structures. The atomistic approach is computationally difficult, especially for large-scaled structures, and therefore the non-classical continuum mechanics are often used

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to capture the size effects. One of the non-classical continuum mechanic approaches to account for the small-scaled effects is the nonlocal elasticity theory (Eringen, 1983), which states the total stress field at any point should depend on the strain field at all points in the whole structures and can account for the size effect of long-range force. Within the framework of the nonlocal continuum mechanics, the size effect of long-range force on the static and dynamic behaviors of small-scaled structures are studied in details by many authors (Daneshmehr, Rajabpoor, & Hadi, 2015; Daneshmehr, Rajabpoor et al., 2014; Ebrahimi & Barati, 2016b; Ebrahimi & Boreiry, 2015; Ghadiri & Shafiei, 2015; Nazemnezhad & Hosseini-Hashemi, 2014; Nejad & Hadi, 2016a; 2016b; Nejad, Hadi, & Rastgoo, 2016; Rahmani & Pedram, 2014; Salehipour, Shahidi, & Nahvi, 2015; Shafiei, Kazemi, & Ghadiri, 2016a; Şimşek & Yurtcu, 2013; Uymaz, 2013). Based on the nonlocal continuum theory, Demir and Civalek (2013) investigated the size effects on the torsional vibration of cylindrical-tubes, Islam, Jia, and Lim (2014) studied the torsional wave propagation and vibration of circular structures, and Arda and Aydogdu (2014) investigated the torsional deformation and vibration of the carbon nanotubes (CNTs) embedded in elastic foundation. These works showed that the torsional wave propagation and vibration can be significantly affected by accounting for the size effect of long-range force.

Alternatively, the size effects on mechanical behaviors can be investigated in terms of gradient elasticity theory, which states that the total stress field must account for additional strain gradient terms to consider microstructural deformation mechanism (Mindlin, 1964; Aifantis, 1992). The gradient elasticity theory modified by Yang, Chong, Lam, and Tong (2002) (known as the modified gradient elasticity or couple stress theory) states that the strain energy density should account for both the strain tensor and the curvature tensor. Within the framework of the modified gradient elasticity theory, the size effect of microstructural deformation mechanism on the static and dynamic behaviors of small-scaled structures are studied in details by many authors (Jabbari, Nejad, & Ghannad, 2015; Khorshidi, Shariati, & Emam, 2016; Nejad & Fatehi, 2015; Rahaeifard, Kahrobaiyan, Ahmadian, & Firoozbakhsh, 2013; Reddy, Romanoff, & Loya, 2016; Shafiei, Mousavi, & Ghadiri, 2016b; Shirazi & Ayatollahi, 2014; Şimşek, Aydın, Yurtcu, & Reddy, 2015; Sourki & Hoseini, 2016; Thai & Kim, 2013). Based on the modified couple stress theory, Gheshlaghi, Hasheminejad et al. (2010) investigated the size-dependent torsional vibration of nano-tubes, Rahaeifard (2015) examined the linear torsional vibration behavior of nano-tubes made of functionally graded FG material , and Setoodeh, Rezaei, and Shahri (2016) studied the nonlinear torsional vibration behavior of nano-tubes. Kahrobaiyan, Tajalli, Movahhedy, Akbari, and Ahmadian (2011) investigated size-dependent torsional deformation and vibration behaviors via the modified gradient elasticity theory. These studies indicated that the microstructure effect potentially plays a important role in studying the size-dependent torsional deformation and torsional vibration behaviors of nano-tubes.

As stated above, the size effects of both long-range force and microstructural deformation mechanism plays a important role in studying the size-dependent torsional deformation and torsional vibration behaviors for structure at small-scale. To understand how the two size effects interact with small-scaled structures, Lim, Zhang, and Reddy (2015) cast the nonlocal elasticity and strain gradient models into a single nonlocal strain gradient theory, whose stress field can account for both non-gradient nonlocal stress field (Eringen, 1983) and pure strain gradient stress field (Aifantis, 1992). It was pointed out by Li, Hu, and Li (2016a) that the nonlocal strain gradient theory can be viewed as merely a combination of the nonlocal elasticity and strain gradient theories, nevertheless the particular combination opens up a novel field of research of modeling and solution approaches which are not available in both the nonlocal elasticity theory and the strain gradient theory. By using a similar constitutive law of the nonlocal elasticity theory, Polyzos et al. (2015) obtained these size-dependent parameters in term of simple lattice dynamics incorporating nonlocal and micro-stiffness effects and explained frequency shift observed in the experimental data for fine-grained material. Li and Hu (2016b); Li, Hu, and Ling (2016b); Tang, Liu, and Zhao (2016) investigated the wave propagating behaviors for CNTs by using beam model and nonlocal strain gradient theory and showed a relatively good matching between the dispersive relation of the atomistic approach and the nonlocal strain gradient model. Owing to the smooth variation of material properties, the FG materials have many advantages in various engineering fields of application, including improved stress spreading, enhanced thermal resistance and enhanced corrosion resistance. Their excellent performances make possible to be applied in small-scaled shaft-type structures by tailoring the nano/micro-scaled material architectures. Within the framework of the nonlocal strain gradient theory, the size effect of both long-range force and microstructural deformation mechanism on the static and dynamic behaviors of small-scaled FG beam-type structures are studied in details by some authors (Li, Hu, & Ling, 2015; Şimşek, 2016; Li, Li, & Hu, 2016c; Ebrahimi & Barati, 2016a; Yang, Yang, & Wang, 2016). However, the size effect of both long-range force and microstructural deformation mechanism on the torsional deformation and vibration of small-scaled functionally graded shaft has not been studied.

To this end, a detailed examination of the nonlocal and microstructure effects on the torsional deformation and vibration behaviors of small-scaled structural systems will be explored in understanding how the two size effects interact with the small-scaled FG shaft. Section 2 of the paper will give a brief overview of nonlocal strain gradient theory that constitutes the basis of this work. The equations of torsional motion and corresponding boundary conditions of the size-dependent FG shaft are derived in Section 3. Then, the analyses of wave propagation problem, free torsional vibration problem, and static torsion problem will be addressed in Sections 4–6, respectively. Finally, some conclusions will be given in Section 7.

2. Brief introduction for nonlocal strain gradient theory

The size-dependent effect on the mechanical behaviors can be observed if the dimension of a structural system becomes comparable to the internal length scale of its material. Under the circumstances, the classical elasticity theory cannot be applied for accurately assessing the mechanical behaviors, and non-classically elasticity theories should be employed. One of these non-classically elasticity theories is the nonlocal strain gradient theory, which can take into account the effects