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Stress analysis of rotating nano-disks of variable thickness made of functionally graded materials

Mohammad Hosseini[a,*], Mohammad Shishesaz[a], Khosro Naderan Tahan[a], Amin Hadi[b]

[a] *Department of Mechanical Engineering, Shahid Chamran University, Ahvaz, Iran*
[b] *Mechanical Engineering Faculty, University of Tehran, Tehran, Iran*

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ABSTRACT

This paper presents the stress analysis of rotating nano-disk made of functionally graded materials with nonlinearly varying thickness based on strain gradient theory. The equilibrium equation and corresponding boundary conditions of nano-disk were obtained using Hamilton's principle. Because of the complexity of governing equations and boundary conditions, the equations are solved using numerical methods. Fixed boundary conditions are considered, in the numerical examples. This analysis is general and can be reduced to classical elasticity. The effect of various parameters such as graded index and thickness profile on stresses and high-order stresses were examined. Values of stresses at inner and outer radial are not zero, because stresses at inner and outer radius accumulate with stresses caused by strain gradient theory. Results show that the effects of thickness parameters are greater than the effect of graded index and the difference between the stress predicted by the classical theory and the strain gradient theory is large when the thickness of nano-disk is small.

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1. Introduction

Nanotechnology is the branch of technology that deals with dimensions and tolerances of less than 100 nanometers, especially the manipulation of individual atoms and molecules. Since the discovery of nanotechnology by Feynman (John, 1997), there are amounts of researches on properties of nano and micro structural elements by adopting experimental investigation, Molecular dynamics simulation and continuum mechanics approach. Both experimental and Molecular dynamics simulation results have shown that the small-scale effects have significant effect on mechanical properties of nano and micro structures (Daneshmehr, Rajabpoor, & Hadi, 2015). Experimental methods in nano scale are very expensive and difficult, molecular dynamic simulations are limited to structures with a small number of molecules and atoms. The continuum mechanics approach is less computationally expensive than molecular dynamic simulations. For solving this barrier, continuum theories are used. Classical continuum mechanics cannot predict small scale effect. Thus, several higher-order continuum theories were suggested to solve this problem. Among these theories, the couple stress theory (Toupin, 1962), Mindlin's strain gradient theory (Mindlin & Eshel, 1968), nonlocal elasticity theory (Eringen, 1972, 1983, 2002), strain gradient theory (Aifantis, 1999), modified couple stress theory (Yang, Chong, Lam, & Tong, 2002) and modified strain gradient

* Corresponding author.
E-mail addresses: s.m.hssini@gmail.com, m-hosseini@phdstu.scu.ac.ir (M. Hosseini).

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theory (Lam, Yang, Chong, Wang, & Tong, 2003) are the most popular and strong ones. Among the size dependent continuum theories, strain gradient theory initiated by Mindlin (Mindlin & Eshel, 1968) has been widely used to analyze many nanostructures problems. According to this theory in addition to the strain tensor, strain gradients are also considered in writing the strain energy density (Danesh & Asghari, 2014).

Many researchers have applied the strain gradient theory for mechanical behavior of nano-sized structures such as beams, plate and shell (Akgöz & Civalek, 2016; Ansari, Gholami, Faghih Shojaei, Mohammadi, & Sahmani, 2015; Ansari, Gholami, & Norouzzadeh, 2016; Gholami, Darvizeh, Ansari, & Sadeghi, 2016; Hosseini & Bahaadini, 2016; Li & Hu, 2016; Li, Hu, & Li, 2016; Ojaghnezhad & Shodja, 2016; shahriari, Karamooz Ravari, & Zeighampour, 2015; Wang, Huang, Zhao, & Zhou, 2016; Zeighampour, 2015; Zeighampour & Tadi Beni, 2015; Zhou, Li, & Wang, 2016).

Rotating disks in size of micron and sub-microns are of practical concern in many tools in micro/nanoelectromechanical systems (MEMS/NEMS), for example micro-gyroscopes (Tsai, Liou, Lin, & Li, 2009, Tsai, Liou, Lin, & Li, 2010, 2011) and micro-motors (Lee, Kim, Bryant, & Ling, 2005). Mechanical behavior of micro-rotating disks based on the strain gradient elasticity is investigated by Danesh and Asghari (Danesh & Asghari, 2014).

Functionally graded materials (FGMs) are a special group of heterogeneous composite materials with mechanical properties changing continuously from one surface to another (Nejad, Rastgoo, & Hadi, 2014b). A number of papers considering various aspects of FGM have been published in recent years (Anani & Rahimi, 2016; Calim, 2016; Golmakaniyoon & Akhlaghi, 2016; Hadi, Rastgoo, Daneshmehr, & Ehsani, 2013; Heydarpour & Aghdam, 2016; Kiełczyński, Szalewski, Balcerzak, & Wieja, 2016; Nejad & Fatehi, 2015; Nejad, Rastgoo, & Hadi, 2014a, 2014b; Şimşek, 2016; Yang, Wang, & Lin, 2016; Zhang & Liew, 2016). Thanks to the advances in technology, FGMs have started to find their ways into micro/nanoelectromechanical systems (MEMS/NEMS) (Kahrobaiyan, Asghari, Rahaeifard, & Ahmadian, 2010; Nejad & Hadi, 2016a, 2016b; Nejad, Hadi, & Rastgoo, 2016; Zhang & Fu, 2012).

Stress analysis of a functionally graded micro/nano rotating disk with variable thickness based on the strain gradient theory is studied by Baghani, Heydarzadeh and Roozbahani (Baghani, Heydarzadeh, & Roozbahani, 2016).

The objective of this study is to obtain the elastic deformations and stresses of rotating nano-disk with nonlinear variable thickness made of functionally graded based on strain gradient theory. There are some problems with derivation of equations and results of above papers (Baghani et al., 2016). Given this problem, in this paper derivation of equations and results have been corrected. In addition, nonlinear function is assumed for profile and properties of disk.

2. Theory and formulation

In this section, the strain gradient theory is formulated for nano-micro discs. In the classic elasticity theory, strain energy density function is dependent on the infinitesimal strain tensor, which is the symmetric part of gradient of displacement field u. In the strain gradient elasticity theory, second gradient of displacement field u appears in the equations of motion and boundary condition equations. The strain gradient elasticity theory introduces dilatation gradient tensor and the deviatoric stretch gradient tensor as well as the symmetric rotation gradient. Strain tensor ε and gradient of strain tensor ξ are:

$$\begin{aligned} \varepsilon &= \frac{1}{2}\left[\nabla u + (\nabla u)^T\right] \\ \xi &= \nabla\varepsilon = \frac{1}{2}\nabla\left[\nabla u + (\nabla u)^T\right] \end{aligned} \tag{1}$$

where

$$\nabla = e_r\frac{\partial}{\partial r} + e_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + e_z\frac{\partial}{\partial z} \tag{2}$$

Therefore radially and the circumferential strains are:

$$\varepsilon = \begin{bmatrix} \frac{\partial u}{\partial r} & 0 & 0 \\ 0 & \frac{u}{r} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

and components of second gradient of displacement field, u are:

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